higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

NOVEMBER EXAMINATION

(10501064)

28 October 2013 (Y-Paper)
13:00–16:00

Non-programmable scientific calculators may be used.

This question paper consists of 6 pages, a 2-page formula sheet and a 2-page answer sheet.

Please turn over

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. QUESTION 3.3.1 and QUESTION 4.5.3 must be done on the attached ANSWER SHEET and hand the ANSWER SHEET in with the ANSWER BOOK.
3. Read ALL the questions carefully.
4. Number the answers according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams and graphs which you have used in determining the answers.
6. If necessary, answers should be rounded off to THREE decimal places, unless stated otherwise.
7. Diagrams are NOT drawn to scale
8. Write neatly and legibly.

QUESTION 1

1.1 Simplify $(2+j)^2-(3-j)^2$ to the form $a+bj$ (3)

1.2 Express $(-1+\sqrt{-3})^2$ in the form $a+bj$ (3)

1.3 Solve for x and y if $(x-4)-(2y+3)j=6+3j$ (3)

1.4 Given: $Z=1-\sqrt{3}j$

1.4.1 Write down the conjugate of Z (1)

1.4.2 Calculate the modulus (r) of $Z=1-\sqrt{3}j$ (1)

1.4.3 Calculate the value of the argument (θ) (2)

1.5 Use De Moivre's theorem to evaluate $\dfrac{(2+5j)^5}{(1-2j)}$ (5)

[18]

QUESTION 2

2.1 Determine $f'(x)$ from first principles if $f(x)=x^2-5x$ (5)

2.2 Use the rules of differentiation to determine the derivatives of the following functions (Leave your answer with a POSITIVE exponent and/or in SURD form):

2.2.1 $f(x)=\sqrt[3]{8x}+\dfrac{8}{x}-ax$ (3)

2.2.2 $y=(2x+3)^5$ (2)

2.2.3 $y=\dfrac{\ln x}{3x-1}$ (3)

2.2.4 $g(x)=2e^{\frac{1}{2}x}\cos x-\tan x$ (3)

2.3 A stone projected vertically upwards with initial velocity $34{,}3\,ms^{-1}$ moves according to the law $s=34{,}3t-4{,}9t^2$; where s is the height of the stone in metres in t seconds.

2.3.1 Calculate the velocity when $t=3$. (2)

2.3.2 Calculate the acceleration of the stone. (2)

2.3.3 Calculate the greatest height reached. (3)

2.3.4 Determine the value(s) of t for which the height of the stone is 29,4 m. (3)

[26]

QUESTION 3

3.1 Determine the following integrals:

3.1.1 $\int(\frac{3}{x}-\frac{x^2}{2}+x^6)dx$ (3)

3.1.2 $\int(e^{3x}+\frac{1}{\cos^2 x}+\frac{\tan x}{\cos x})dx$ (4)

3.2 Evaluate $\int_1^3\left(\frac{x^5+x^2}{x^3}\right)dx$ (3)

3.3 Given: $f(x)=x^2-1$

3.3.1 Sketch the curve $f(x)=x^2-1$ on the attached ANSWER SHEET and clearly shade the region bounded by f and the lines, $x=1$ and $x=2$. (5)

3.3.2 Calculate the magnitude of the area of the shaded region using integration (3)

[18]

QUESTION 4

4.1 4.1.1 Solve for k: $k+\frac{16}{k}=10$ (2)

4.1.2 Hence, or otherwise solve for x:

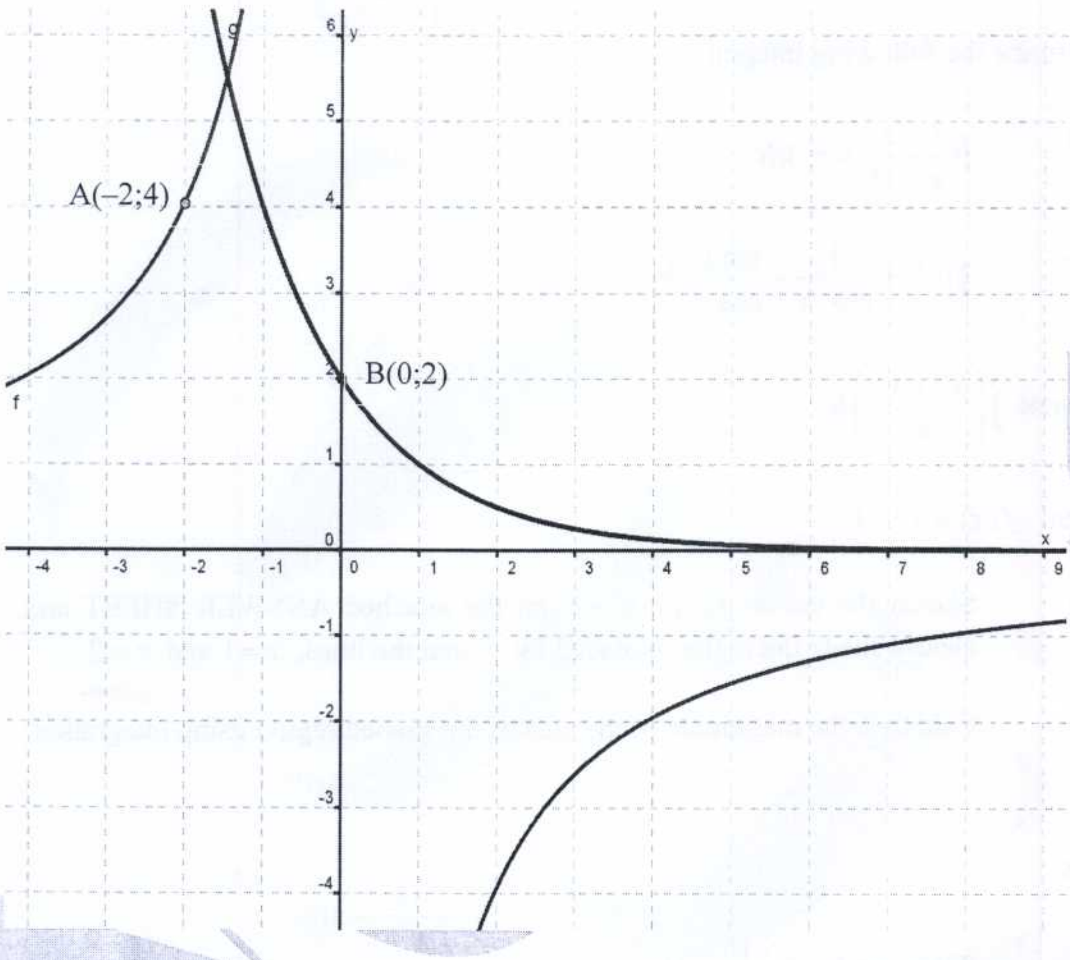
$$x^2+x+\frac{16}{x^2+x}=10$$ (4)

4.2 If $ax^2+bx+c=0$ has roots $\frac{-1\pm\sqrt{1+8}}{4}$, determine possible values of a, b and c. (4)

4.3 Find the constants p and r so that the polynomial x^3+px+r has a remainder -9 when it is divided by $(x+1)$ and a remainder of -1 when it is divided by $(x-1)$. (5)

4.4 The diagram below shows the graphs of the functions $f(x) = 2^{-x+p}$ and $g(x) = \frac{q}{x}$.

The point A(–2; 4) lies on g. The graph of f cuts the y axis at B(0; 2).



4.4.1 Calculate the values of p and q (3)

4.4.2 Write down the domain of $g(x)$ (1)

4.4.3 Write down the range of $f(x)$ (1)

4.4.4 What is the equation of the asymptote of $f(x) = 2^{-x+p}$. Deduce the asymptote of $h(x) = f(x) + 1$? (2)

4.4.5 Write down the equations of the axes of symmetry of $g(x)$ (2)

4.5 A patient is required to take at least 18 grams of protein, 6 milligrams of vitamin C and 5 milligrams of iron per meal, which consists of two types of food, A and B. Type A contains 9 grams protein, 2 milligrams of vitamin C and no iron per mass unit. Type B contains 3 grams of protein, 2 milligrams of vitamin C and 5 milligrams of iron mass per mass unit. The energy value of A is 800 kilojoules and that of B is 400 kilojoules per mass unit. A patient is not allowed to have more than 4 mass units of A and 5 mass units of B. There are x mass units of A and y mass units of B on the patient's plate.

	FOOD A	FOOD B
Protein	9	3
Vitamin C	2	2
Iron	0	5

Use the above information to answer the following questions:

4.5.1 Write down ALL the constraints with respect to the above information in terms of x and y. (5)

4.5.2 What is the kilojoules intake per meal? (1)

4.5.3 Represent the constraints graphically on the graph paper on the attached ANSWER SHEET. (6)

4.5.4 Deduce, from the graphs, the value of x and y which give the minimum kilojoule intake per meal for the patient. (2)

[38]

TOTAL: 100

~~B(0;2)~~

(-2; 4) $g(x) = \frac{9}{x}$

B(0;2) $f(x) = 2^{-x-p}$

$y = 2^{-x-p}$

$2 = 2^{-0-p}$

$\frac{2}{1} = \frac{1p}{1}$

$2 = p$

$gx = \frac{9}{x}$

$y = \frac{9}{x}$

$\frac{4}{1} = \frac{9}{-2}$

$-8 = 9$

D = x

R = y

Domain $g(x) = (-\infty; \infty)$

Range $f(x) = (-\infty; \infty)$

4.4.4 $f(x) = 2^{-x+p}$

$f(1) = 2^{-1+p}$

FORMULA SHEET

1. $Z = r\cos\theta + r\,j\sin\theta$

2. $Z = a \pm bj$ or $Z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r \lfloor\underline{\theta} = r \ cis \ \theta$

4. $m = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

5. $\dfrac{d}{dx}x^n = nx^{n-1}$

6. $If \quad y = ka^x = ka^x Ina$

7. $\dfrac{d}{dx}k = 0$

8. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}[f(x).g(x)] = f(x)\ g'(x) + f'(x)\ g(x)$

9. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

10. $\dfrac{dy}{dx} = \dfrac{du}{dx}\times\dfrac{dy}{du}$ or $\dfrac{d}{dx}f(g(x)) = f'(g(c))\ g'(x)$

11. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

12. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or If $f(x) = e^x$ then $f'(x) = e^x$

13. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

14. *If* $y = \sin x$ *then* $\dfrac{dy}{dx} = \cos x$ or *If* $f(x) = \sin x$ *then* $f'(x) = \cos x$

15. *If* $y = \cos x$ *then* $\dfrac{dy}{dx} = -\sin x$ *or If* $f(x) = \cos x$ *then* $f'(x) = -\sin x$

16. *If* $y = \tan x$ *then* $\dfrac{dy}{dx} = \sec^2 x$ *or If* $f(x) = \tan x$ *then* $f'(x) = \sec^2 x$

17. $If \quad y = \cot x \quad then \ \frac{dy}{dx} = -\cos ec^2 x$ $\quad$ $or \ If \ \ f(x) = \cot x \ \ then \ \ f^{'}(x) = -\cos ec^2 x$

18. $If \ \ y = \sec x \ \ then \ \frac{dy}{dx} = \sec x \tan x$ $\quad$ $or \ If \ \ f(x) = \sec x \ \ then \ \ f^{'}(x) = \sec x \tan x$

19. $If \ \ y = \cos ecx \ \ then \ \frac{dy}{dx} = -\cos ecx \cot x$ $\quad$ $or \ If \ \ f(x) = \cos ecx \ \ then \ \ f^{'}(x) = \cos ecx \cot x$

20. $If \ \ y = In \sec x \ \ then \ \frac{dy}{dx} = \tan x \ x \ If \ \ f(x) = In \sec x \ \ then \ \ f^{'}(x) = \tan x$

21. $\int x^n \, dx \; = \; \frac{x^{n+1}}{n+1} + c$

22. $\int k\, x^n \, dx \; = \; k \int x^n \, dx$ where k is a constant value.

23. $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

24. $\int \frac{k}{x}\, dx = \; k \; \ln x + c$

25. $\int k a^x \, dx \quad = k\, a^x \log_a e + c$

26. $\int \; e^{kx} \; dx \; = \frac{e^{kx}}{k} + c$

27. $\int \sin x.dx = -\cos x + c$

28. $\int \cos x dx = -\sin x + c$

29. $\int \tan x.dx = In \sec x + c$

30. $[\, f(x) \,]_a^b = f(b) - f(a)$