higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

NOVEMBER 2012

(10501064)

29 October (Y-Paper)
13:00 – 16:00

Non-programmable scientific calculators may be used.

This question paper consists of 6 pages, a 2-page formula sheet and 2 answer sheets.

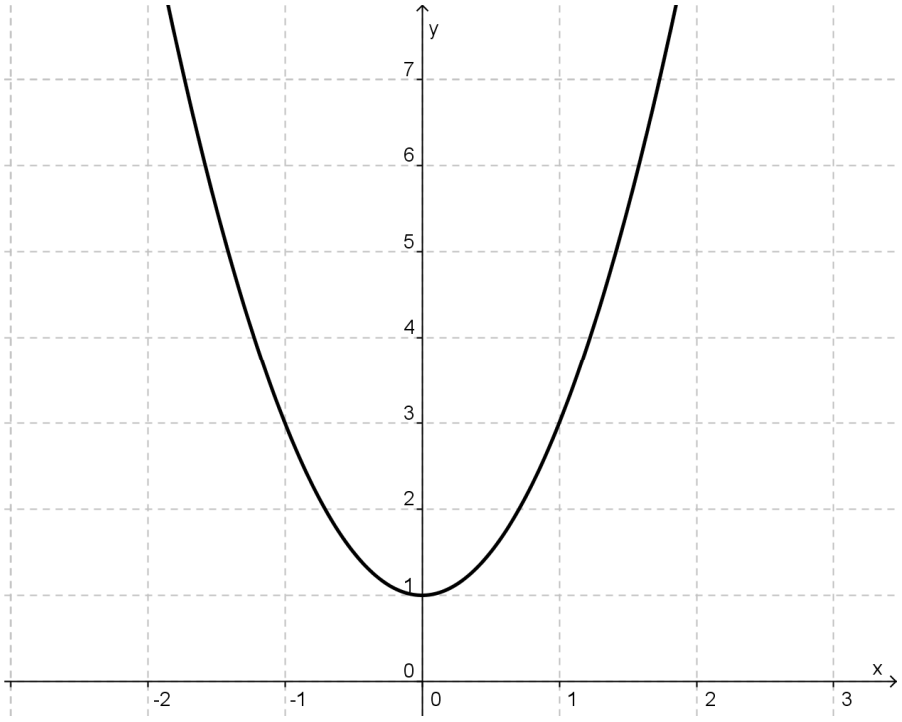
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc, which you have used in determining the answers.
5. If necessary, answers should be rounded off to THREE decimal places, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Write neatly and legibly.

QUESTION 1

1.1 The diagram given below represents the graph $f(x) = 2x^2 + q$. The graph cuts the y-axis at (0 ; 1).



1.1.1 Determine the value of q (1)

1.1.2 Is the graph of $y = 2x^2 + q$ a function or a non-function? (1)

1.1.3 Determine the equation of the inverse of the function $y = 2x^2 + q$ (2)

1.1.4 Sketch the inverse of the function $y = 2x^2 + q$ (3)

1.2 The polynomial $f(x) = ax^3 - x^2 - 4x + b$ is divisible by $x - 2$ but leaves a remainder of 6 when it is divided by $(x + 1)$

Use the factor/remainder theorem to determine the values of a and b (4)

1.3 A carpenter makes two types of tables, A and B

- He has 400 m^2 floor space available
- Each table A requires 30 m^2 and each table B requires 40 m^2 of floor space
- He does not have enough skilled labourers to make more than 8 tables of type A and 6 of type B
- According to public demand he has to make at most two type B tables for each type A table

Let x be the number of tables of type A and y the number of tables of type B

1.3.1 Write the constraints with respect to the above information in terms of x and y (4)

1.3.2 Represent graphically ALL the constraint inequalities on ANNEXURE A (attached) and clearly indicate the feasible region (6)

1.3.3 If the profit on each table is the same, determine by means of a search line, how many of each type of table should be made for maximum profit (2)

[23]

QUESTION 2

2.1 Find the derivative of $f(x) = -3x^2 + 4$ by using first principles (5)

2.2 Determine the derivatives of the following. Leave answers with positive exponent and in surd form where applicable.

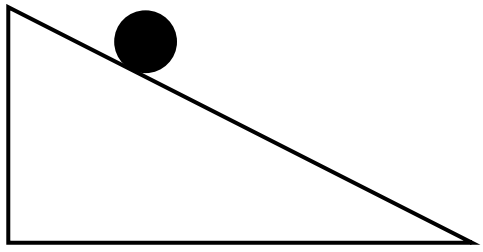
2.2.1 $f(x) = \frac{1}{x} - \frac{3x^2 + 4x^5}{x^4} + 2a$ (where 'a' is a constant) (4)

2.2.2 $f(x) = 3x^3.e^x$ (3)

2.2.3 $f(x) = \ln\ (4x - 5)$ (2)

2.2.4 $f(x) = \frac{x^2 - 3x}{x^3 + 1}$ (3)

2.3 A ball rolls down an inclined plane. The distance S (in centimetres) that it rolls in t seconds is given by the equation $S = t^3 + 3t^2 + 3$ where $0 \leq t \leq 3$



2.3.1 Determine the velocity of the ball at $t = 1\frac{1}{2}$ seconds (3)

2.3.2 What is the maximum velocity of the ball (2)

2.3.3 At what time is the velocity 24 cm/s (3)

[25]

QUESTION 3

3.1 Find the integrals of the following:

3.1.1 $\int (2x-3)(1-2x)\,dx$ (3)

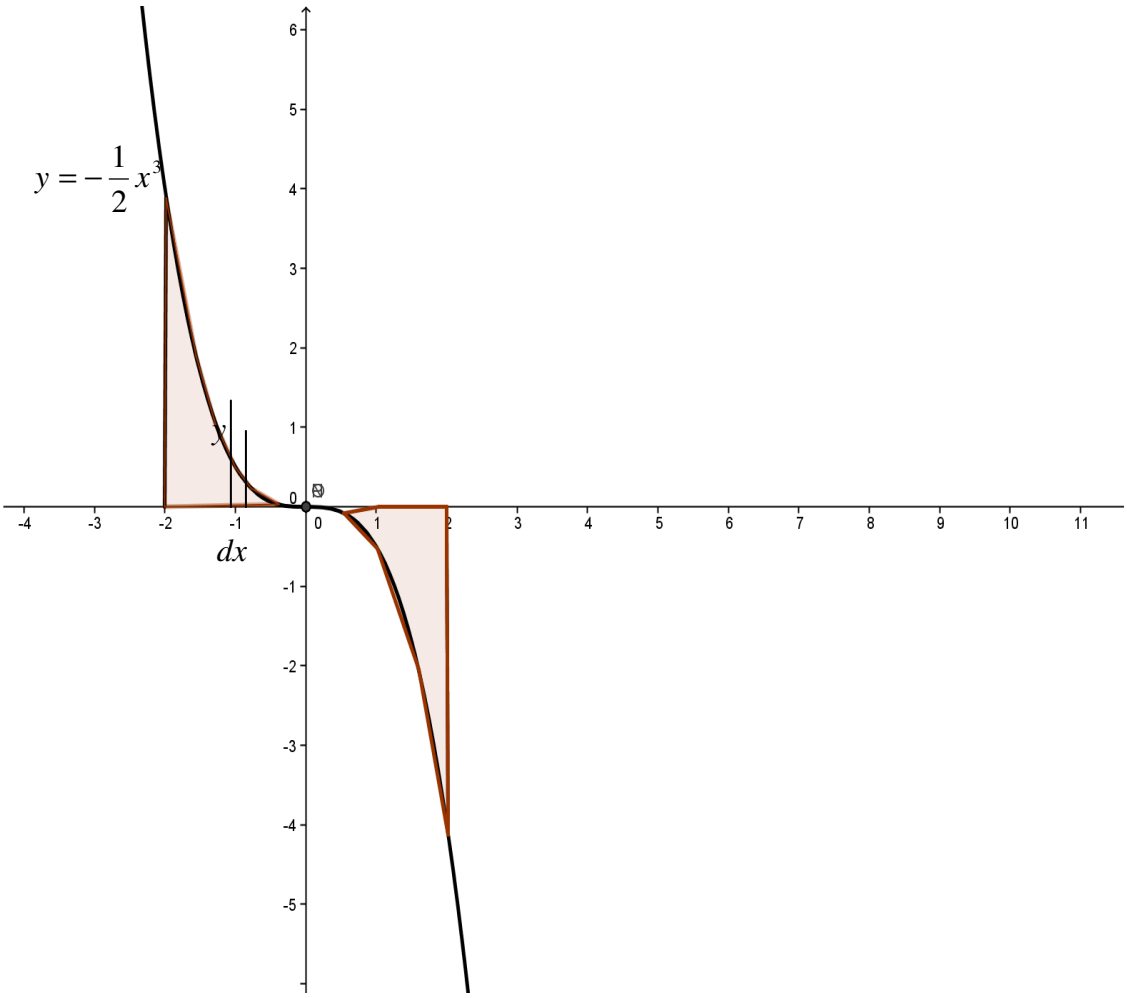
3.1.2 $\int \left(\frac{2}{x^2} - 2\sqrt{x} + \frac{3}{e^{-x}}\right) dx$ (3)

3.1.3 $\int \left(3e^{2x} - \frac{4}{x}\right) dx$ (3)

3.2 Evaluate the expression $\int_1^3 (\frac{1}{3}x^3 - 2x^2 + 3)dx$ (4)

3.3 Given below is the graph of $y = -\frac{1}{2}x^3$

Determine the area of the shaded region



(5)

[18]

QUESTION 4

4.1 During the last medical examination on 5 members of an old age facility the following results were obtained. The TABLE shows the age of the participant and the heart rate per minute. Use the information to answer the following questions:

Age	Heart rate per minute
60	78
74	80
67	68
43	66
56	37

4.1.1 Use the information above to construct a scatter plot on ANNEXURE B (attached) with the ages on the *x*-axis and heart rates on the *y*-axis. (5)

4.1.2 Find the sample regression equation by the method of least squares. (7)

4.1.3 Predict the heart rate of a 68-year old. (1)

4.2 Find the standard deviation of the following set of scores.

8	2	10	14	16

(6)

4.3 In a basket there are three red and seven blue flowers. In a second basket there are four red and six blue flowers. A flower is selected at random from each basket. Represent the above information in a tree diagram. (7)

4.4 Use the tree diagram from QUESTION 4.3 to answer the questions.

4.4.1 Determine the probability that the flower selected from the first basket is red and the flower selected from the second basket is blue. (2)

4.4.2 Determine the probability that the two flowers are different in colour. (3)

4.4.3 Determine the probability that the flower selected from the second basket is blue. (3)

[34]

TOTAL: 100

FORMULA SHEET

1. $m = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

2. $\dfrac{d}{dx} x^n = nx^{n-1}$

3. $\dfrac{d}{dx} k = 0$

4. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}[f(x).g(x)] = f(x)\ g'(x) + f'(x)\ g(x)$

5. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

6. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{\mathrm{d}}{\mathrm{dx}} \mathrm{f}\left(\mathrm{g}(x)\right) = \mathrm{f}'\left(\mathrm{g}(x)\right)\ \mathrm{g}'(x)$

7. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

8. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or If $f(x) = e^x$ then $f'(x) = e^x$

9. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

10. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11. $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c$

12. $\int k\, x^n\, dx = k \int x^n\, dx$ where k is a constant value.

13. $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

14. $\int \dfrac{k}{x}\, dx = k \ln x + c$

15. $\int e^{kx}\, dx = \dfrac{e^{kx}}{k} + c$

16. $\left[f(x) \right]_a^b = f(b) - f(a)$

17. $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

18. variance = $s^2 = \dfrac{\sum(x_i - \bar{x})^2}{n}$

19. standard deviation = $s = \sqrt{\dfrac{\sum(x_i - \bar{x})^2}{n}}$

20. $\hat{y} = a + bx$

21 $b = \dfrac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$ or $b = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

22. $a = \bar{y} - b\bar{x}$

23. $P(A) = \dfrac{n(A)}{n(S)}$

24. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

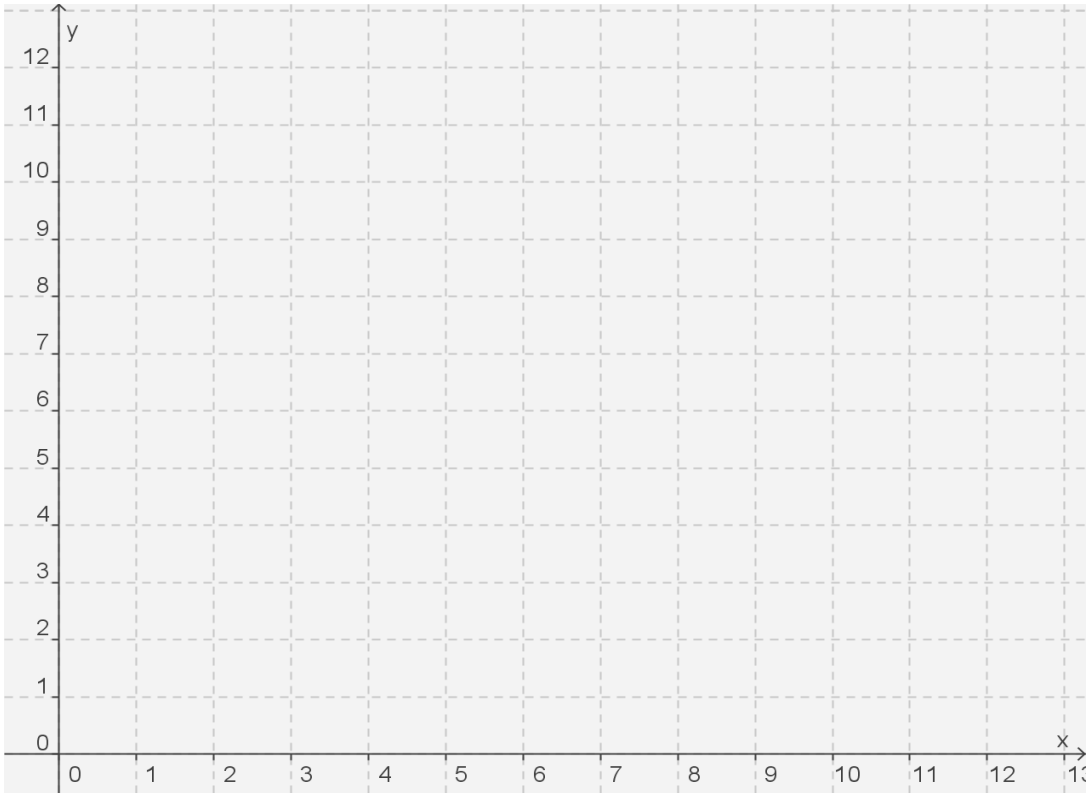
25. $P(A/B) = \dfrac{P(A \& B)}{P(B)}$

ANSWER SHEET **EXAMINATION NUMBER:**

ANNEXURE A

Detach ANNEXURE A and hand it in with your ANSWER BOOK.

1.3.2



ANSWER SHEET **EXAMINATION NUMBER:**

Detach ANNEXURE B and hand it in with your ANSWER BOOK.

ANNEXURE B

4.1 4.1.1