higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

NOVEMBER 2010

MATHEMATICS
(Second Paper)
NQF LEVEL 4

4 NOVEMBER 2010

This marking guideline consists of 9 pages.

✓= 1 MARK √ $=\frac{1}{2}$ MARK

QUESTION 1

1.1 $OB = 100mm$ (radius)

$$\frac{AC}{AB} = \frac{MO}{BO}$$

$$AC = \frac{AB \times MO}{BO}$$

$$AC = \frac{200 \times 60}{100}$$

$$AC = 120mm$$

Alternative methods

$OB = 100mm$ (radius)

$OM = 60mm$

$MB^2 = OB^2 - OM^2$

$= 100^2 - 60^2$

$= 6400$

$MB = \sqrt{6400}$

$= 80mm$

$BC = 160mm$

$AC^2 = AB^2 - BC^2$

$= 200^2 - 160^2$

$= 14400$

$AC = \sqrt{14400}$

$= 120mm$ ✓

Simple solution:

$MB = MC$ because $OM \perp BC$ ✓

Also $MO \,//\, CA$ ✓

$\therefore MO = ½\, AC$ ✓

$\therefore AC = 2MO = 2(60) = 120mm$ ✓

Another solution, using similar triangles:

Since $\Delta BMO \,///\, \Delta BCA$ ✓

$$\frac{AC}{AB} = \frac{MO}{BO}$$ ✓

But $AC = 2BO$ radii ✓

$$\therefore \frac{\text{AC}}{\text{2BO}} = \frac{MO}{BO}$$

$\therefore AC = 2MO = 120mm$ ✓ (4)

1.2 $x = 80°$ ✓ (corresponding angles) DE // HI
$y = 80°$ ✓ (alternate angles) or opp angles of a parm
$\hat{HIG} = 180° - (80° + 40°) = 60°$ (angles in a triangle)
$z = 60°$ ✓ (alternate angles) (3)

1.3 $\hat{O}_1 = 80°$ ✓ (angles in the centre , twice angle at circumference)
$\hat{Q}_4 = \hat{R}_{5\&6} = \frac{180° - 80°}{2} = 50°$✓ (isosceles triangle) ✓
$\hat{R}_7 = \hat{O}_1 = 80°$ ✓ (alternate angles) OQ//SR (4)

1.4 1.4.1 $A = 65^0$ ✓ Sum of angles of a triangles✓
$B = 65^0$ ✓ (angles sub by the same chord)
$\hat{C}_2 = A = 65$✓ Given (4)

1.4.2 $E\hat{D}C = 180° - 65° = 115°$ ✓ (opposite angles of a cyclic quad) ✓
$\hat{D} + \hat{C}_2 = 115° + 65° = 180°$ ✓
$\hat{D}$ *and* $\hat{C}_2$ are co-interior angles ✓
$\therefore ED \| AC$

Alternative Method

$\hat{E}_1 + \hat{E}_2 + \hat{C}_2 = 180^0$ (opposite angles of a cyclic quad) ✓
$E_2 = 180^0 - 25^0 - 65^0$
$\hat{E}_2 = 90^0$
$\therefore ED \| AC$✓ ($\hat{E}$, equal and alternate to $\hat{F}_1$)

(4)

1.5 1.5.1

$B\hat{T}F = T\hat{E}F$ tan - chord theorem ✓

$B\hat{T}F = T\hat{C}D$ tan - chord theorem ✓

$\therefore T\hat{E}F = T\hat{C}D$

$\therefore EF//CD$ Corresp.angles equal ✓ (3)

1.5.2

$D\hat{P}F = P\hat{F}E$ EF//CD alt angles ✓

$= P\hat{T}C$ ✓

$D\hat{P}F = P\hat{T}D$ tan - chord theorem ✓

$\therefore P\hat{T}C = P\hat{T}D$ ✓

$\therefore TP$ bisects CTD (4)

1.6 1.6.1

$x^2 + y^2 = r^2$

$r^2 = 25 + 9$

$r^2 = 34$ ✓

Equation : $x^2 + y^2 = 34$ ✓ (2)

1.6.2

$$M_{OR} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - (-3)}{0 - (5)}$$

$$= -\frac{3}{5}$$ ✓

$$M_{AB} = \frac{5}{3}$$ ✓

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{5}{3}(x - 5)$ ✓

$3y + 9 = 5(x - 5)$

$3y + 9 = 5x - 25$

$-5x + 3y + 9 + 25 = 0$

$-5x + 3y + 34 = 0$ ✓ (4)

[32]

QUESTION 2

2.1

$\sqrt{-1}(\sqrt{-1}+\sqrt{9}-\sqrt{-16}$
$=i(i+3-\sqrt{16}.\sqrt{-1})$ √
$=i(i+3-4i)$ √
$=i^2+3i-4i^2$ √
$=1+3i+4$ √
$=3i+3$
$=3\sqrt{-1}+3$ ✓

Alternate:

$\sqrt{-1}\left[\sqrt{-1}+3-4\sqrt{-1}\right]$ ✓
$=+\sqrt{-1}\left[-3\sqrt{-1}+3\right]$
$=-3(-1)+3\sqrt{-1}$ ✓
$=3+3\sqrt{-1}$ ✓

(3)

2.2

$(-3+5i)(-2-2i)^2$
$=(-3+5i)(4+8i+4i^2)$ ✓
$=(-3+5i)(4+8i+4.-1)$ ✓
$=(-3+5i)(8i)$
$=-24i+40i^2$ ✓
$=-40-24i$ ✓

(4)

2.3.

$\left(\frac{8\,cis\,60°}{4\,cis\,30°}\right)^4\times\left(\frac{6\,cis\,50°}{12\,cis\,240°}\right)^{-2}$
$=\left(\frac{8\,cis\,60°}{4\,cis\,30°}\right)^4\times\left(\frac{12\,cis\,240°}{6\,cis\,50°}\right)^2$ ✓
$=(2\,cis\,30°)^4\times(2\,cis\,190°)^2$ ✓ ✓
$=16\,cis\,120°\times 4\,cis\,380°$ ✓
$=64\,cis\,500°$ ✓
$=64\,cis\,140°$

Alternate:

$=(2\,cis\,30°)^4\times(2\,cis\,190°)^{-2}$ ✓
$=16\,cis\,120°\times 4\,cis\,380°$ ✓ ✓
$=16\,cis\,120°\times 4\,cis\,20°$ ✓
$=64\,cis\,140°$ ✓

(5)

2.4 2.4.1

$2x+3yi=10-12i$
$2x=10$
$x=5$ ✓

$3yi=-12i$ ✓
$3y=-12$
$y=-4$ ✓

(3)

2.4.2 $(3-4i)^2 = \frac{x+iy}{i^2}$

$9 - 24i + 16i^2 = \frac{x+yi}{-1}$ ✓

$-9+24i-16(-1) = x+yi$ ✓

$7+24i = x+yi$ ✓

$x=7$ ✓

$y=24$ ✓ (5)

[20]

QUESTION 3

3.1 $\cos 2\alpha$

$=\cos(\alpha+\alpha)$ ✓

$=\cos\alpha\cos\alpha - \sin\alpha\sin\alpha$ ✓

$=\cos^2\alpha - \sin^2\alpha$ ✓ (3)

3.2 $\sin(\alpha+\beta)=\sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ ✓

$\sin(45°+30°)=\sin 45°.\cos 30° + \cos 45°.\sin 30°$ ✓

$=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}.\frac{1}{2}$ ✓

$=\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$ ✓

$=\frac{\sqrt{6}+\sqrt{2}}{4}$ ✓

$=0{,}966$ ✓ (6)

3.3 3.3.1 $\sin^2\theta=1$

$\sin\theta=\sin\theta=\pm\sqrt{1}$ ✓

$\sin\theta = \sin\theta=\pm 1$ ✓

$\theta=90°$

$= 1{,}57$ *radians* ✓

or $\theta=270°$

$=4{,}712$ *radians* ✓ (4)

3.3.2 $3\cos 2\theta + \cos\theta + 1 = 0$

$3(2\cos^2\theta - 1) + \cos\theta + 1 = 0$ ✓

$6\cos^2\theta + \cos\theta - 2 = 0$ ✓

$(2\cos\theta - 1)(3\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$ ✓ *or* $\cos\theta = -\frac{2}{3}$ ✓

$\theta_1 = 60°$ $\theta_3 = 131{,}81°$

$= 1{,}047$ *radians* ✓ $= 2{,}3$ *radians* ✓

$\theta_2 = 300°$ $\theta_4 = 228{,}19°$

$= 5{,}236$ *radians* ✓ $= 3{,}982$ *radians* ✓ (9)

3.4 $\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$

LHS:

$= \frac{\cos\theta.\sin 2\theta - \sin\theta.\cos 2\theta}{\sin\theta\cos\theta}$ ✓

$= \frac{\cos\theta(2\sin\theta\cos\theta) - \sin\theta(1 - 2\sin^2\theta)}{\sin\theta\cos\theta}$ ✓

$= \frac{2\sin\theta\cos^2\theta - \sin\theta(1 - 2\sin^2\theta)}{\sin\theta\cos\theta}$ ✓

$= \frac{2\sin\theta(1 - \sin^2\theta) - \sin\theta(1 - 2\sin^2\theta)}{\sin\theta\cos\theta}$ ✓

$= \frac{\sin\theta\,[2(1 - \sin^2\theta) - (1 - 2\sin^2\theta)]}{\sin\theta\cos\theta}$ ✓

$= \frac{2 - 2\sin^2\theta - 1 + 2\sin^2\theta}{\cos\theta}$ ✓

$= \frac{1}{\cos\theta}$

$= \sec\theta \quad = RHS$ ✓

Alternate:

✓

$= \frac{2\sin\theta\cos\theta}{\sin\theta} - \frac{2\cos^2\theta - 1}{\cos\theta}$ ✓

$= \frac{2\cos\theta}{1}$ ✓ $- \frac{2\cos^2\theta - 1}{\cos\theta}$

$= \frac{2\cos^2\theta}{\cos\theta}$ ✓ $- \frac{2\cos^2\theta - 1}{\cos\theta}$

$= \frac{2\cos^2\theta - 2\cos^2\theta + 1}{\cos\theta}$ ✓

$= \frac{1}{\cos\theta}$ ✓

$= \sec\theta$ ✓ (7)

3.5.1 $A\hat{B}G = x$ (alternate angles)

$C\hat{A}B = y - x$ ✓

$A\hat{C}B = 180° - [y - x) + (x + w]\ldots$ $\angle s\ of\ a\ \Delta.$ √

$= 180° - y + x - x - w$ √

$= 180° - y - w$ √

$= 180° - (w + y)$ √ (4)

3.5.2 $\dfrac{AC}{\sin(x+w)} = \dfrac{AB}{\sin(180° - (y+w))}$ √

$\dfrac{AC}{\sin(x+w)} = \dfrac{AB}{\sin(w+y)}$ ✓

$AC = \dfrac{AB\sin(w+x)}{\sin(w+y)}$ ✓ (3)

3.5.3 $AC = \dfrac{AB\ \sin(w+x)}{\sin(w+y)}$ ✓

$= \dfrac{800\sin(50° + 23°)}{\sin(50° + 54°)}$

$= 788{,}465\ m$ ✓ (2)

[38]

QUESTION 4

4.1 R 26 000 × 12 = R 312 000 ✓

R 62 710 + 35% of (312000 – 270000) ✓

R 62710 + R14 700

= R 77 410 ✓

Less Rebates:

R 77410 - R 8280 - R 5040 ✓

= R 64 090

Monthly salary : R 312 000 - R 64 090

= 247 910 ÷ 12

= R 20 659,17 ✓ (5)

4.2.1 $A = P(1+i)^n$

$= 8(1+0{,}075)^3$ ✓

$= R9{,}94$ ✓ (2)

4.2.2 $A = P(1+i)^n$

$8 = P(1+0{,}075)^2$ ✓

$P = \dfrac{8}{(1+0{,}075)^2}$ ✓

$= R6{,}92$ ✓ (3)

[10]

TOTAL: 100