higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

NOVEMBER 2010

(10501064)

1 November (Y-Paper)
13:00 – 16:00

REQUIREMENTS: Graph paper

Candidates may use scientific calculators.

This question paper consists of 6 pages and a 3-page formula sheet.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams and graphs, which you have used in determining the answers.
5. Answers should be rounded off to THREE decimal places, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Write neatly and legibly.

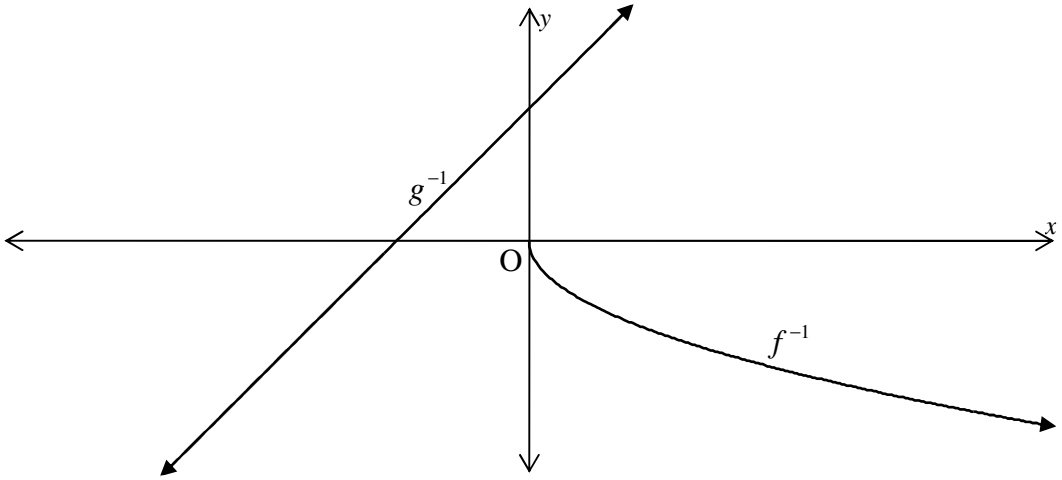
QUESTION 1

1.1 The polynomial $f(x) = 2x^3 - 5x^2 + x + a$ has $2x + 1$ as one of its factors.

1.1.1 Determine the value of the constant a. (3)

1.1.2 Use the factor theorem or any other method to factorize $f(x)$ completely. (3)

1.2 The graphs of the inverse functions, f^{-1} and g^{-1}, of $f(x) = x^2; x \leq 0$ and $g(x) = x - 2$ respectively, are shown below:



1.2.1 Write down the range of f^{-1}. (1)

1.2.2 Determine the equation of f^{-1}. (1)

1.2.3 Write down the equation of g^{-1} (1)

1.2.4 Determine algebraically the points of intersection of the graphs of f and g^{-1}. (4)

1.3 A factory manufactures fax machines and scanners. On any given day x fax machines and y scanners are produced. Constraints such as labour, time, cost of materials and demand influence the production and profit. These constraints are represented by the following inequalities:

$$3x + 2y \leq 24;\ x + 2y \leq 12;\ x + y \geq 6;\ x + 2y \geq 8;\ x > 0;\ y > 0$$

1.3.1 Using a scale of 10 mm = 1 unit make a graphical representation of the above constraints, and clearly indicate the feasible region. (8)

1.3.2 If the profit P, in rands, is given by the objective function $y = -\frac{4}{5}x + \frac{P}{200}$, determine the profit on each fax machine and on each scanner. (3)

1.3.3 Use a search line to determine how may fax machines and scanners should be produced per day so as to yield a maximum profit? (3)

1.3.4 Calculate the maximum profit. (2)
[30]

QUESTION 2

2.1 Determine $f'(x)$ from first principles if $f(x)=2x^3$. (6)

2.2 Determine the derivatives of the following functions:

2.2.1 $y=(1-2x)^3$ (2)

2.2.2 $f(x)=\dfrac{2x^3}{\sqrt{x}}+10$ (2)

2.2.3 $y=x\ln x$ (2)

2.2.4 $y=\dfrac{e^{2x}+1}{2x^2-3}$ (2)

2.3 An object is blasted into the ground such that it reaches a depth given by the formula, $f(t)=-\sqrt{t}$ metres, where t is the time in seconds.

2.3.1 What is the velocity of the object at $t=1$ second. (2)

2.3.2 Determine the acceleration of the object at $t=\dfrac{1}{4}$. [Hint: Acceleration is the rate of change of velocity.] (3)
[30]

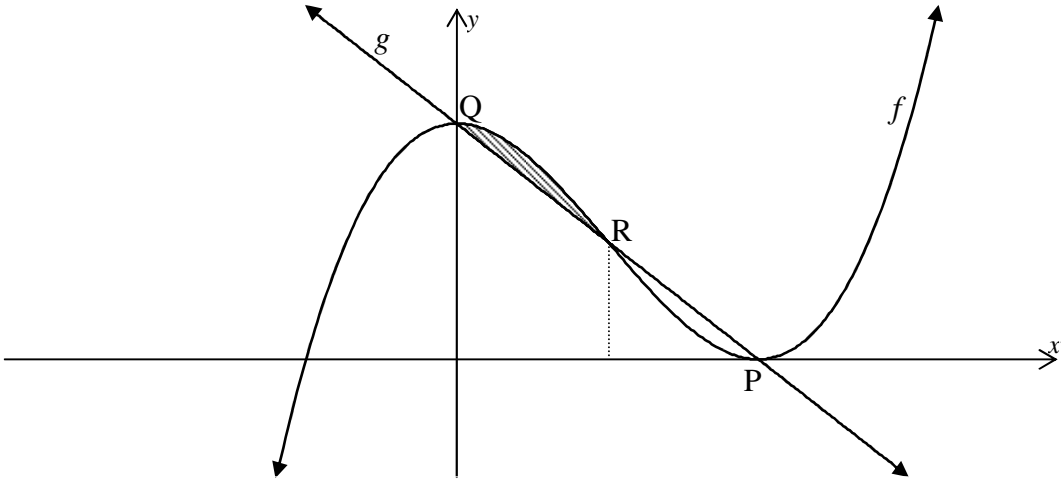
QUESTION 3

3.1 Determine the following indefinite integrals:

3.1.1 $\int 4x(x-1)(x+1)dx$ (2)

3.1.2 $\int\left(6e^{2x}+\dfrac{2}{x}\right)dx$ (3)

3.2 The diagram below shows the graphs of the functions $f(x)=x^3-3x^2+c$ and $g(x)=mx+c$, where m and c are constants. P and Q are turning points of f with P on the x axis and Q on the y axis. R is a point of inflection of f. The graph of g passes through points P, Q and R.



3.2.1 Show that the coordinates of P are (2;0). (3)

3.2.2 Calculate the value of c. (2)

3.2.3 Write down the coordinates of Q. (1)

3.2.4 Determine the equation of g. (2)

3.2.5 Calculate the coordinates of R. (2)

3.2.6 Calculate the area under the curve of f between $x=0$ and $x=1$. (3)

3.2.7 Hence or otherwise determine the area of the shaded region. (2)

[20]

QUESTION 4

4.1 Ms Khambule randomly selects a group of 5 learners in her class and compares their reading scores against their scores in a standardized mathematics test. The scores are listed below:

Learner	**1**	**2**	**3**	**4**	**5**
Reading %	65	90	55	50	75
Mathematics	60	85	50	50	65

4.1.1 Construct a scatter plot for the above data showing the Reading scores on the horizontal axis and the Mathematics scores on the vertical axis. Use a scale of 1 cm = 10%. (3)

4.1.2 Show the line of best fit on the scatter plot. (1)

4.1.3 Complete the following table and enter the values of A, B, C, D, E and F in your ANSWER BOOK:

Learner	Reading % x	Mathematics % y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$
1	65	60				
2	90	85				
3	55	50				
4	50	50				
5	75	65				
	$\sum x = A$	$\sum y = C$			$\sum (x-\bar{x})(y-\bar{y}) = E$	$\sum (x-\bar{x})^2 = F$
	$\bar{x} = B$	$\bar{y} = D$				

4.1.4 Use the information in the above table to determine the regression equation by the method of least squares. [Round off the values of the coefficients to TWO decimal places.] (4)

4.1.5 Dumi is a learner in Ms Khambule's class, who was not part of the sample. Use the regression equation to estimate Dumi's Mathematics score (to the nearest percent) if her Reading score was 45%. (2)

4.2 Alfie takes both Mathematics and Physical Science. Past results indicate that the probability of passing Mathematics is 0,75, the probability of failing Physical Science is 0,35, and that of passing at least one of the two subjects is 0,85. Calculate the probability that Alfie will:

4.2.1 pass Physical Science (2)
4.2.2 pass both subjects (2)
4.2.3 fail both subjects (2)
4.2.4 pass exactly one of the two subjects. (2)

4.3 Miriam has four coins in her purse. one coin is fair, one has two tails, the third has two heads and the fourth is weighted in such a way that the probability of obtaining a head is ⅓ (one third). If Miriam randomly selects a coin from her purse and tosses it, what is the probability that it will land tails up? (6)

[30]

TOTAL: 100

FORMULA SHEET

1. $m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2. $\frac{d}{dx} x^n = nx^{n-1}$

3. $\frac{d}{dx} k = 0$

4. $\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$ or $\frac{d}{dx}[f(x).g(x)] = f(x)\ g'(x) + f'(x)\ g(x)$

5. $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ or $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

6. $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$ or $\frac{d}{dx} f\left(g(x)\right) = f'\left(g(c)\right)\ g'(x)$

7. If $y = \ln kx$ then $\frac{dy}{dx} = \frac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \frac{k}{kx}$

8. If $y = e^x$ then $\frac{dy}{dx} = e^x$ or If $f(x) = e^x$ then $f'(x) = e^x$

9. If $y = e^{kx}$ then $\frac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

10. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

FORMULA SHEET (Continued)

11. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

12. $\int k\,x^n\,dx = k\int x^n\,dx$ where k is a constant value.

13. $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$

14. $\int \frac{k}{x}\,dx = k\,\ln x + c$

15. $\int e^{kx}\,dx = \frac{e^{kx}}{k} + c$

16. $\left[f(x)\right]_a^b = f(b) - f(a)$

17. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

18. variance $= s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

19. Standard deviation $= s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

20. $\hat{y} = a + bx$

21. $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$ or $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

FORMULA SHEET (Continued)

22. $a = \bar{y} - b\bar{x}$

23. $P(A) = \dfrac{n(A)}{n(S)}$

24. $P(A \text{ or } B) = P(A) + P(B)$

25. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

26. $P(A/B) = \dfrac{P(A \& B)}{P(B)}$