education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 4

NOVEMBER 2009

(10501064)

2 November (X-Paper)
09:00 – 12:00

REQUIREMENTS: **Graph paper**

Scientific calculators may be used.

This question paper consists of 7 pages and a 2-page formula sheet.

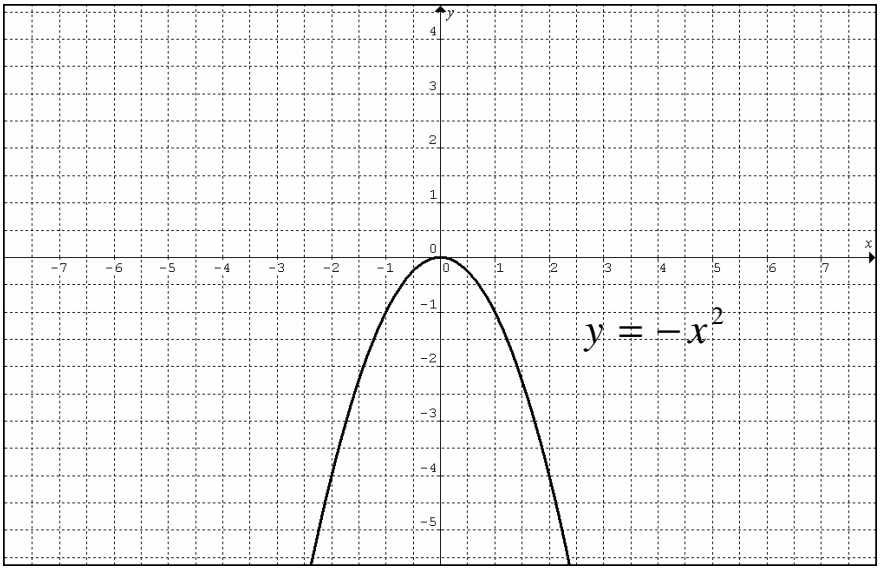
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
5. Where necessary, answers should be rounded off to THREE decimal places, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Write neatly and legibly.

QUESTION 1

1.1 The diagram given below represents the graph of $y = -x^2$.



1.1.1 Write down the equation of the axis of symmetry for the graph of $y = -x^2$ (1)

1.1.2 Is the graph of $y = -x^2$ a function? Give a reason. (1)

1.1.3 Determine the equation of the inverse of the function $y = -x^2,\ x \geq 0$ and write down its domain. (2)

1.1.4 Sketch the inverse of the function $y = -x^2, x \geq 0$. (3)

1.1.5 Is the inverse function continuous or discontinuous? (1)

1.2 Calculate the value of p if $x+3$ is a factor of $f(x) = 2x^3 + x^2 - 5x - p$. (3)

1.3 A group of students intend selling x hamburgers and y chicken burgers at a soccer match. They have meat for 300 hamburgers and 400 chicken burgers at most. Each burger is sold in a polystyrene container, of which 500 are available. The students suspect that hamburgers will be at most twice as popular as the chicken burgers.

1.3.1 Two of the constraints are $x \geq 0$ and $y \geq 0$. Write down FOUR other constraints in respect of the above information in terms of x and y. (4)

1.3.2 Using a scale of 1 cm = 100 burgers, represent the inequalities graphically and indicate the feasible region. (5)

1.3.3 The students make a profit of R4,50 on each hamburger sold and R3,00 on each chicken burger sold. Write the equation that represents the total profit in terms of x and y. (1)

1.3.4 Use the search line method to determine how many of each burger should be sold to obtain the maximum profit. Hence determine the maximum profit. (3)

[24]

QUESTION 2

2.1 Determine $f'(x)$ from first principles if $f(x)=\frac{1}{x}$ (5)

2.2 Determine the derivatives of the following:

2.2.1 $y=\frac{2}{\sqrt{x^3}}+2x^{-1}-2t$, where t is a constant (4)

2.2.2 $f(x)=\frac{x^2+1}{\ln x}$ (2)

2.2.3 $y=e^{2x-1}$ (3)

2.3 Find the equation of the tangent to the curve $y=(2x-3)(1-x)$ at $x=2$ (4)

2.4 A body moves in a straight line from A to B where it comes to rest. After t seconds its distance from A, in metres, is given by: $s = 108\ t + 13{,}5\ t^2 - t^3$.
Determine:

2.4.1 The velocity of the body after 5 seconds (3)

2.4.2 The acceleration of the body after 5 seconds (3)

[24]

QUESTION 3

3.1 Find the integrals of the following:

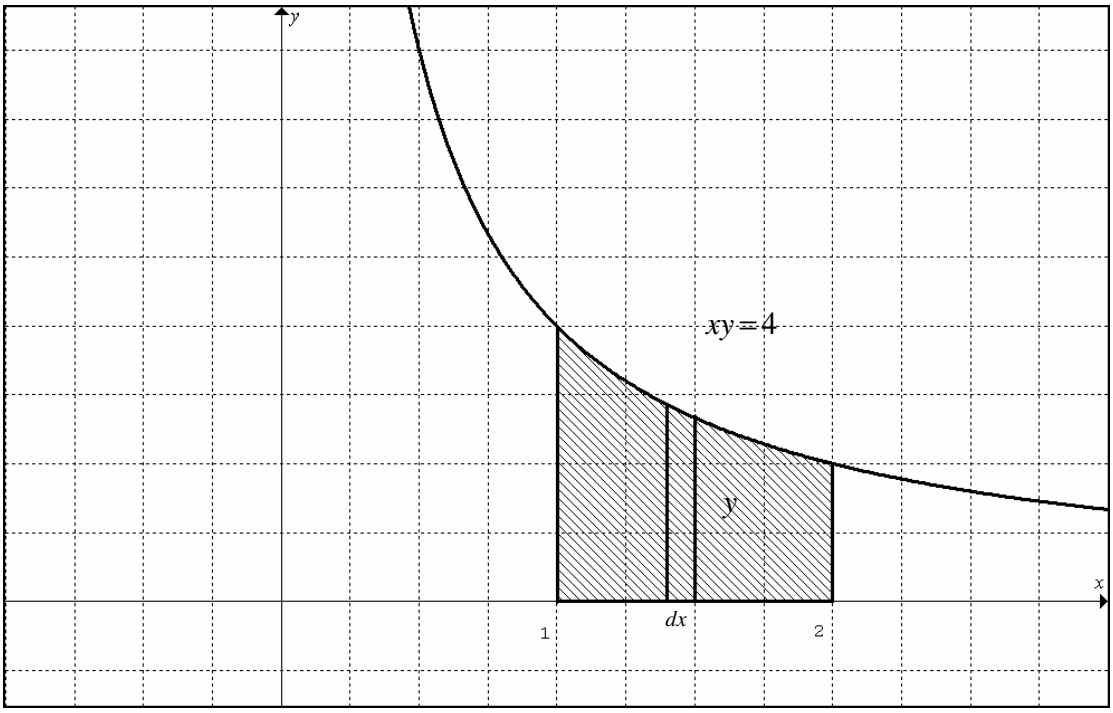
3.1.1 $\int (1-2x)^2\,dx$ (2)

3.1.2 $\int\left(e^{3x}+\frac{1}{2}\cos 2x-3\sin 6x\right)dx$ (3)

3.2 Evaluate:

$$\int_3^5 (-x^2+8x)\,dx$$ (2)

3.3 Given below is the graph of the hyperbola $xy=4$. Determine the area of the shaded part of the graph.



(5)

[12]

QUESTION 4

4.1 As a manager in a factory you have observed 10 workers on the shop floor, timing how long it takes each of them to assemble a product. It was possible to match these times with the length of the workers' experience. The results obtained are shown below.

EMPLOYEE	EXPERIENCE x (IN MONTHS)	TIME y (MINUTES)
A	2	27
B	5	26
C	3	30
D	8	20
E	5	22
F	9	20
G	12	16
H	16	15
I	1	30
J	6	19

4.1.1 Construct a scatter plot for the above data with the employees' experience on the horizontal axis and the time to assemble the product on the vertical axis. (6)

4.1.2 Find the sample regression equation by the method of least squares. (8)

4.1.3 Predict the time it would take an employee with 4 months' experience to assemble the product. (2)

4.2 The ages of 7 second-hand cars are as follows:

13; 7; 10; 15; 12; 18; 9

Calculate the standard deviation of this data. Show ALL workings. (7)

4.3 There are 3 green balls, 5 red balls and 2 yellow balls in a bag. You take out one without looking. What is the probability of picking out a yellow ball? (2)

4.4 Zandile rolls a pair of dice during a game.

4.4.1 Determine the probability that Zandile would throw a double. (2)

4.4.2 Determine the probability that she would throw a score of 7. (2)

4.5 A fair coin is tossed three times. What is the probability of obtaining three heads? (2)

4.6 A fair coin is tossed and a fair dice is rolled.

4.6.1 Show this event on a tree diagram. (4)

4.6.2 Determine the probability of obtaining a head and a 6. (2)

4.6.3 Determine the probability of obtaining a tail and an even number (3)
[40]

TOTAL: 100

FORMULAE SHEET

1. $m = \underset{h\to 0}{Lim}\dfrac{f(x+h)-f(x)}{h}$

2. $\dfrac{d}{dx}x^n = nx^{n-1}$

3. $\dfrac{d}{dx}k = 0$

4. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}\left[f(x).g(x)\right] = f(x)\ g'(x) + f'(x)\ g(x)$

5. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

6. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{d}{dx}f\left(g(x)\right) = f'\left(g(c)\right)\ g'(x)$

7. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{kx}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{kx}$

8. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or . If $f(x) = e^x$ then $f'(x) = e^x$

9. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

10. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c$

12. $\int k\,x^n\,dx = k\int x^n\,dx$ where k is a constant value.

13. $\int [f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$

14. $\int \dfrac{k}{x}\,dx = k\ln x + c$

15. $\int \sin kx\,dx = \dfrac{-\cos kx}{k} + c = -\dfrac{1}{k}\cos kx + c$

16. $\int \cos kx\,dx = \dfrac{\sin kx}{k} + c = \dfrac{1}{k}\sin kx + c$

17. $\int \sec^2 kx\ dx = \frac{1}{k}\tan kx + c$

18. $\int \cos ec^2 kx\ dx = -\frac{1}{k}\cot kx + c$

19. $\int \sec kx \tan kx\ dx = \frac{1}{k}\sec kx + c$

20. $\int \cot kx \cos ec\ kx\ dx = -\frac{1}{k}\cos ec\ kx + c$

21. $\left[f(x) \right]_a^b = f(b) - f(a)$

22. $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

23. variance $= s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

24. standard deviation $= s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

25. $\hat{y} = a + bx$

26. $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$ or $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

27. $a = \bar{y} - b\bar{x}$

28. $P(A) = \frac{n(A)}{n(S)}$

29. $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$