higher education & training

Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 3

NOVEMBER EXAMINATION

(10501053)

29 October 2013 (X-Paper)
09:00–12:00

REQUIREMENTS: **Graph paper**

Scientific calculators may be used.

This question paper consists of 6 pages, 1 formula sheet and 2 annexures.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Write neatly and legibly.

QUESTION 1

1.1 Determine $f'(x)$ from the first principles if $f(x) = -5x^2 + 1$. (4)

1.2 Determine the following:

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 3x}$$ (2)

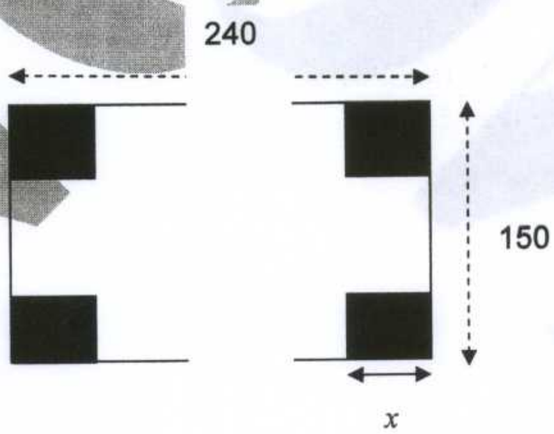
1.3 Use differentiation rules to determine $\frac{dy}{dx}$ of the following. (Leave your answer with POSITIVE exponent and in SURD form where applicable).

1.3.1 $y = \left(\frac{6x^4 - x}{3x^2} + 5 \right)$ (3)

1.3.2 $y = \sqrt{x} + \frac{3}{x^3}$ (3)

1.3.3 $y = \sqrt[3]{x^2} + \frac{1}{4x^4}$ (3)

1.4 A sheet of aluminum 240 mm by 150 mm has square edges of side x millimeters cut out. The edges are then folded up so as to form a box x millimeters high without a lid.
NOTE: Diagram is not drawn to scale



Hint: $V = l \times b \times h$
$V = (240 - 2x)(150 - 2x)(x)$

1.4.1 Find x so that the volume of the box is a maximum. (3)

1.4.2 Calculate the maximum volume. (2)

[20]

QUESTION 2

2.1 Without the use of a calculator and without converting to polar form, simplify the given complex numbers to standard form of $a+bi$:

2.1.1 $\left(3-\sqrt{-16}\right)\left(5+\sqrt{-16}\right)$ (3)

2.1.2 $(-6+3i)(-6-3i)$ (2)

2.1.3 $i(2+3i)+(-2-i)$ (3)

2.2 Given $z=-3-5i$

2.2.1 Write down the conjugate of z. (1)

2.2.2 Calculate the modulus (r) and argument (θ) of $\bar{z}$. (4)

2.2.3 Express $\bar{z}$ in polar form. (1)

2.2.4 Explain the relationship between z and $\bar{z}$. (1)

2.2.5 Represent z and $\bar{z}$ graphically in an Argand diagram. (4)

2.3 Simplify the following complex numbers to standard form:

2.3.1 $\dfrac{\left(2+\sqrt{3}i\right)\left(3-\sqrt{3}i\right)}{2-\sqrt{3}i}$ (5)

2.3.2 $\dfrac{1{,}2(\cos 212+i\sin 212)}{2{,}4(\cos 162+i\sin 162)}$ (3)

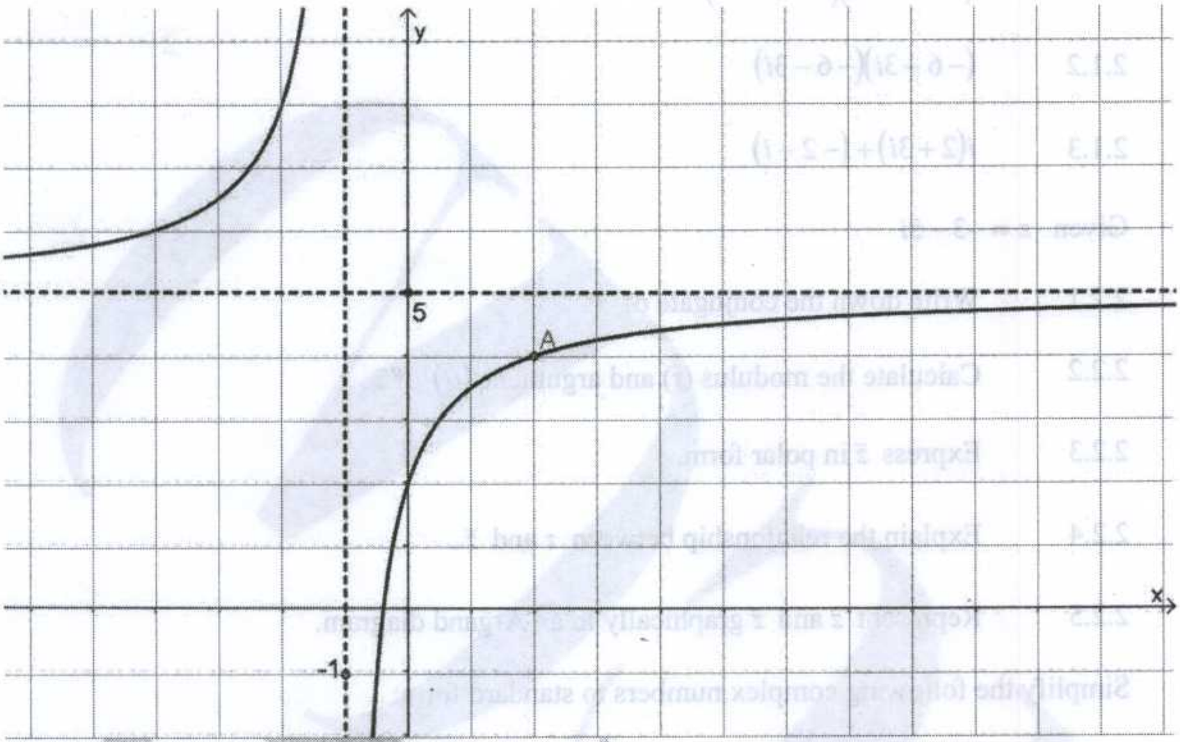
2.3.3 $4\,\underline{|120^0} \;+\; 6\,\underline{|315^0}$ (3)

[30]

QUESTION 3

3.1 The diagram below represents the graph of:

$f(x)=\dfrac{a}{x-p}+q$ where $A(2,4)$ is a point on the graph



3.1.1 Determine the values of a, p and q. (4)

3.1.2 Write down the range of $f(x)$. (1)

3.2 Given: $y=-\frac{1}{2}x^2+2x+4\frac{1}{4}$. Write the function in the form:

3.2.1 $y=a(x-p)^2+q$ (4)

3.2.2 Determine the x and y intercepts of the function. (4)

3.2.3 Sketch the graph of $y=-\frac{1}{2}x^2+2x+4\frac{1}{4}$ (Use ANNEXURE A to answer QUESTION 3.2.3) (4)

3.2.4 Write down the range of $y=-\frac{1}{2}x^2+2x+4\frac{1}{4}$ (1)

3.2.5 Is the graph continuous or discontinuous? (1)

3.3 A particular industrial problem reduces to the following system of inequalities:

$10 \le x \le 40$
$30y + 10x \ge 300$
$10y + 10x \le 450$
$0 \le y \le 30$

3.3.1 Draw the graphs with the given constraints. (Use ANNEXURE B to answer QUESTION 3.3.1). (5)

3.3.2 Determine the feasible region. (1)

3.3.3 Find the values of x and y which will maximise $P = 0{,}2x + 0{,}3y$ and the maximum profit. (4)

3.4 Solve for x:

$(2x+1)^2 - 4(x-1)^2 < 0$ (5)

3.5 Solve for x by using a quadratic formula:

$2x^2 + 5 + 20x = 20 - 8x^2 + x$ (4)

3.6 Solve for x and y:

$y = 5x - 26$
$y = x^2 - 7x + 10$ (5)

3.7 Simplify the following:

$$\frac{xy - y^2}{x^2 - y^2} \times \frac{x^2 + xy}{y}$$ (3)

3.8 Prove that:

$$\left(x + y^{-1}\right)^{-1} = \frac{y}{xy + 1}$$ (4)

[50]

TOTAL: 100

2 = 18
3 = 23
4 = 9
6 = 6
7 -
9 -

1 4 | -4
1 3 | 3

1 9 | -9
1 4 | -4
13

30
-43

FORMULA SHEET

1. $z = r\cos\theta + r\,j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ where $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \bar{z}}$

4. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

5. $r\,\underline{|\theta} = r\ cis\ \theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x-p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \frac{a}{(x+p)} + q$

11. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

12. $\frac{d}{dx}x^n = nx^{n-1}$

13. $\frac{d}{dx}k = 0$

14. $Dx[kf(x)] = kDx[f(x)]$

15. $Dx[f(x) \pm g(x)] = Dx[f(x)] \pm Dx[g(x)]$