higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

NOVEMBER EXAMINATION

MATHEMATICS
(First Paper)
NQF LEVEL 3

29 OCTOBER 2013

This marking guideline consists of 11 pages.

✓ = 1 MARK √ = ½ MARK

QUESTION 1

1.1 $f(x) = -5x^2 + 1$

$f(x+h) = -5(x+h)^2 + 1$ ✓ (For correct substitution)

$$f'(x) = \lim_{h\to 0} \frac{-5x^2 - 10xh - 5h^2 + 1 - (-5x^2 + 1)}{h}$$ ✓ Expanding their $f(x+h)$ correctly

$$f'(x) = \lim_{h\to 0} \frac{-5x^2 - 10xh - 5h^2 + 1 + 5x^2 - 1}{h}$$ Multiplication

$$f'(x) = \lim_{h\to 0} \frac{-10xh - 5h^2}{h}$$ Addition

$$f'(x) = \lim_{h\to 0} \frac{h(-10x - 5h)}{h}$$ ✓ Take out common factor

$f'(x) = -10x - 5(0)$

$f'(x) = -10x$ ✓ (Answer only – one mark) (4)

1.2 $$\lim_{x\to 3} \frac{x^2 - 9}{x^2 - 3x}$$

$$\lim_{x\to 3} \frac{(x-3)(x+3)}{x(x-3)}$$ √ √

$$\lim_{x\to 3} \frac{(x+3)}{x}$$ √

$$\frac{3+3}{3}$$

$= 2$ √ (2)

1.3 1.3.1

$$y = \left(\frac{6x^4 - x}{3x^2} + 5\right)$$

$$y = \frac{6x^4}{3x^2} - \frac{x}{3x^2} + 5$$
$$\surd \quad \surd$$
$$y = 2x^2 - \frac{1}{3}x^{-1} + 5$$
$$\surd \quad \surd \quad \surd$$
$$y' = 4x + \frac{1}{3}x^{-2} + 0$$

$$y' = 4x + \frac{1}{3x^2}\surd \qquad (3)$$

1.3.2

$$y = \sqrt{x} + \frac{3}{x^3}$$
$$\surd \quad \surd$$
$$y = x^{\frac{1}{2}} + 3x^{-3}$$
$$\surd \quad \surd$$
$$y' = \frac{1}{2}x^{\frac{-1}{2}} - 9x^{-4}$$

$$\surd \quad \surd$$
$$y' = \frac{1}{2\sqrt{x}} - \frac{9}{x^4} \qquad (3)$$

1.3.3

$$y = \sqrt[3]{x^2} + \frac{1}{4x^4}$$
$$\surd \quad \surd$$
$$y = x^{\frac{2}{3}} + \frac{1}{4}x^{-4}$$
$$\surd \quad \surd$$
$$y' = \frac{2}{3}x^{\frac{-1}{3}} - x^{-5}$$
$$\surd \quad \surd$$
$$y' = \frac{2}{3\sqrt[3]{x}} - \frac{1}{x^5} \qquad (3)$$

1.4 1.4.1 $V = l \times b \times h$

$V = (240 - 2x)(150 - 2x)(x)$

$V = x(36\ 000 - 780x + 4x^2)$

$V = 36\ 000x - 780x^2 + 4x^3$ √

√ √ √

$\frac{dv}{dx} = 12x^2 - 1\ 560x + 36\ 000$ (Carry forward mistakes)

$12x^2 - 1\ 560x + 36\,000 = 0$ √

$x^2 - 130x + 3000 = 0$

$(x - 30)(x - 100) = 0$

$x = 30$ / $x = 100$ Both answers

When $x = 30\,mm$ √ the volume is maximum (3)

1.4.2 $v = 36000(30) - 780(30)^2 + 4(30)^2$ ✓

✓ (Carry forward mistake) (2)

$v = 486000mm^3$

[20]

QUESTION 2

2.1 2.1.1 $(3 - \sqrt{-16})(5 + \sqrt{-16})$

√ √

$(3 - 4i)(5 + 4i)$

$15 + 12i - 20i - 16i^2$ √

√

$15 + 12i - 20i + 16$

$31 - 8i$

√ √

(3)

2.1.2 $(-6 + 3i)(-6 - 3i)$

√ √

$36 + 18i - 18i - 9i^2$

√ √

$45 + 0i$ Full marks if answer is 45 only. (2)

2.1.3 $i(2+3i)+(-2-i)$
√ √ √
$2i+3i^2-2-i$
√
$2i-3-2-i$

√ √
$-5+i$

(3)

2.2 2.2.1 $\bar{z}=-3+5i$ ✓ (Conjugate) (1)

2.2.2 $r=\sqrt{(-3)^2+(5)}$ ✓

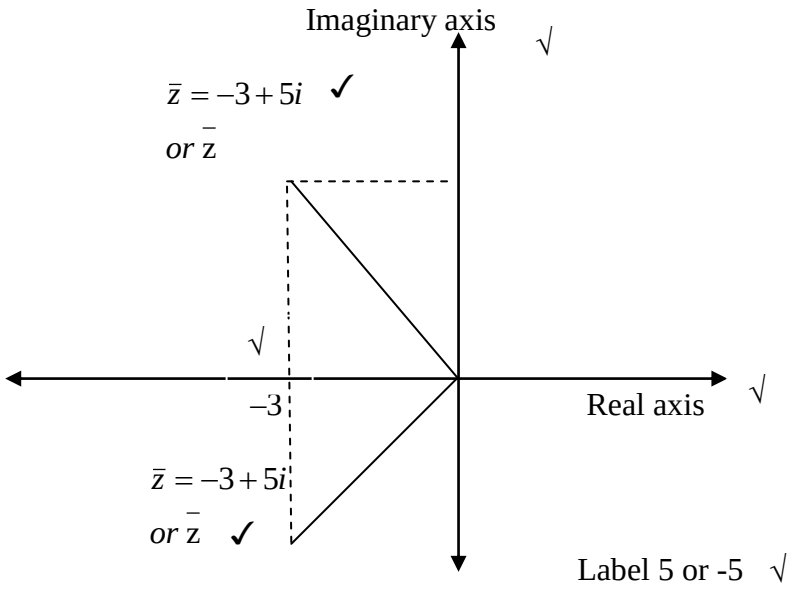
$r=\sqrt{34}$ ✓
$r=5{,}831$
$\theta=\tan^{-1}\left(\frac{b}{a}\right)$
$\theta=\tan^{-1}\left(\frac{5}{-3}\right)$
$\theta=-59{,}036$

$\theta=180°-59{,}036$ ✓(For recognizing the reference angle)
$\theta=120{,}964°$ ✓
(4)

2.2.3 $5{,}831\angle 120{,}964°$ ✓ or $5{,}831 cis 120{,}964^0$ ✓ (Carry forward mistakes) (1)

2.2.4 Conjugates ✓ (Accept any reasonable answer e.g. The imaginary sign is changed/ product is real) (1)

2.2.5



(4)

2.3 2.3.1

$$\frac{(2+i\sqrt{3})(3-i\sqrt{3})}{2-i\sqrt{3}}$$

√ √

$$\frac{6-i2\sqrt{3}+i3\sqrt{3}-3i^2}{2-i\sqrt{3}}$$

✓

$$\frac{9+\sqrt{3}i}{2-i\sqrt{3}}\times\frac{2+i\sqrt{3}}{2+i\sqrt{3}}$$ ✓

$$\frac{18+i9\sqrt{3}+i2\sqrt{3}+3i^2}{4+i2\sqrt{3}-i2\sqrt{3}-3i^2}$$ √ √

√ √

$$\frac{15}{7}+\frac{i11\sqrt{3}}{7}$$ (5)

2.3.2

$$\frac{1{,}2(\cos 212+i\sin 212)}{2{,}4(\cos 162+i\sin 162)}$$

$$\frac{1{,}2cis\,212}{2{,}4cis\,162}$$ √ √ (Do not penalize for working in rectangular form)

√ √

$0{,}5cis\,50$

$0{,}5(\cos 50+i\sin 50)$

√ √

$0{,}321+0{,}383i$ (3)

2.3.3

$4\angle 120^0 + 6\angle 315^0$

$4(\cos 120+i\sin 120)+6(\cos 315+i\sin 315)$

√ √ √ √

$-2+3{,}464i+4{,}244-4{,}244i$

√ √

$2{,}244-0{,}779i$ (3)

[30]

QUESTION 3

3.1 3.1.1 $p = -1$ ✓
$q = 5$ ✓
$y = \frac{a}{2+1} + 5$ at $(2,4)$

$\sqrt{4} = \frac{a}{2+1} + 5$ (For substituting *x* and *y)*
$\sqrt{}$

$a = -3$ ✓ (4)

3.1.2 $y \varepsilon R$ and $y \neq 5$ ✓ (Accept any correct notation) (1)

3.2 3.2.1 ✓
$y = -\frac{1}{2}(x^2 - 4x) + \frac{17}{4}$

✓
$y = -\frac{1}{2}(x^2 - 4x + 4 - 4) + \frac{17}{4}$

✓
$y = -\frac{1}{2}(x-2)^2 + 2 + \frac{17}{4}$

$y = -\frac{1}{2}(x-2)^2 + \frac{25}{4}$ ✓ (Give full marks for answer only) (4)

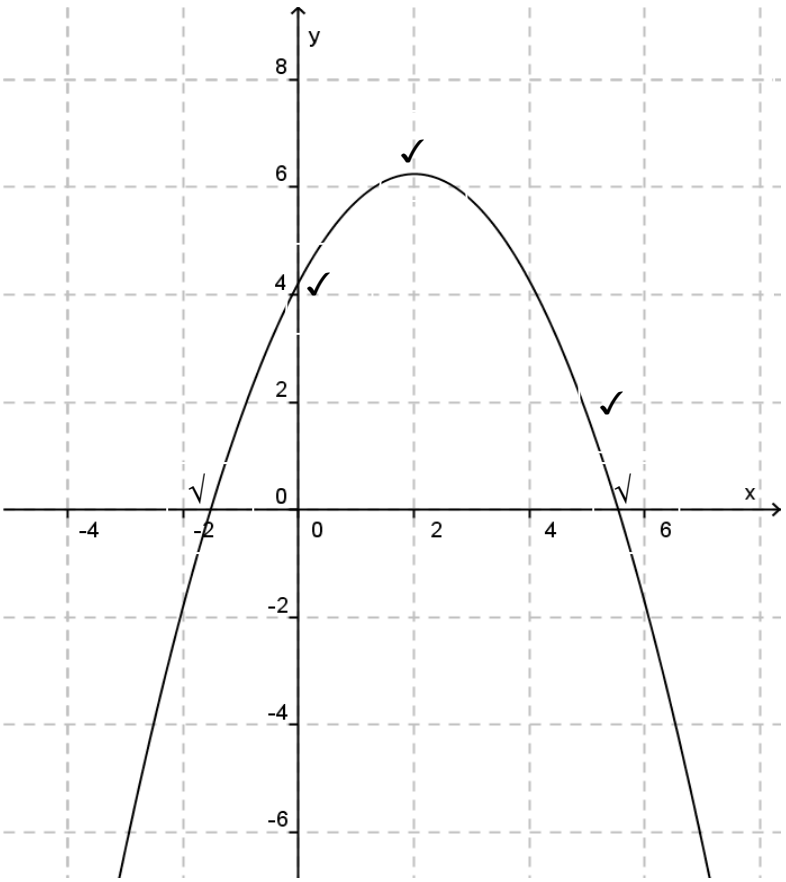
3.2.2 $y = -\frac{1}{2}x^2 + 2x + \frac{17}{4}$
Let $x = 0 \ \therefore y = \frac{17}{4}$ ✓
Let $y = 0$
$0 = -\frac{1}{2}x^2 + 2x + \frac{17}{4}$

$0 = -2x^2 + 8x + 17$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-2)(17)}}{(2)(-2)}$ ✓ or $x = \frac{-2 \pm \sqrt{(2)^2 - 4\left(-\frac{1}{2}\right)\left(\frac{17}{4}\right)}}{2\left(-\frac{1}{2}\right)}$

$x = \frac{-8 \pm \sqrt{200}}{-4}$
✓ ✓
$x = -1{,}536 \qquad x = 5{,}536$ (4)

3.2.3



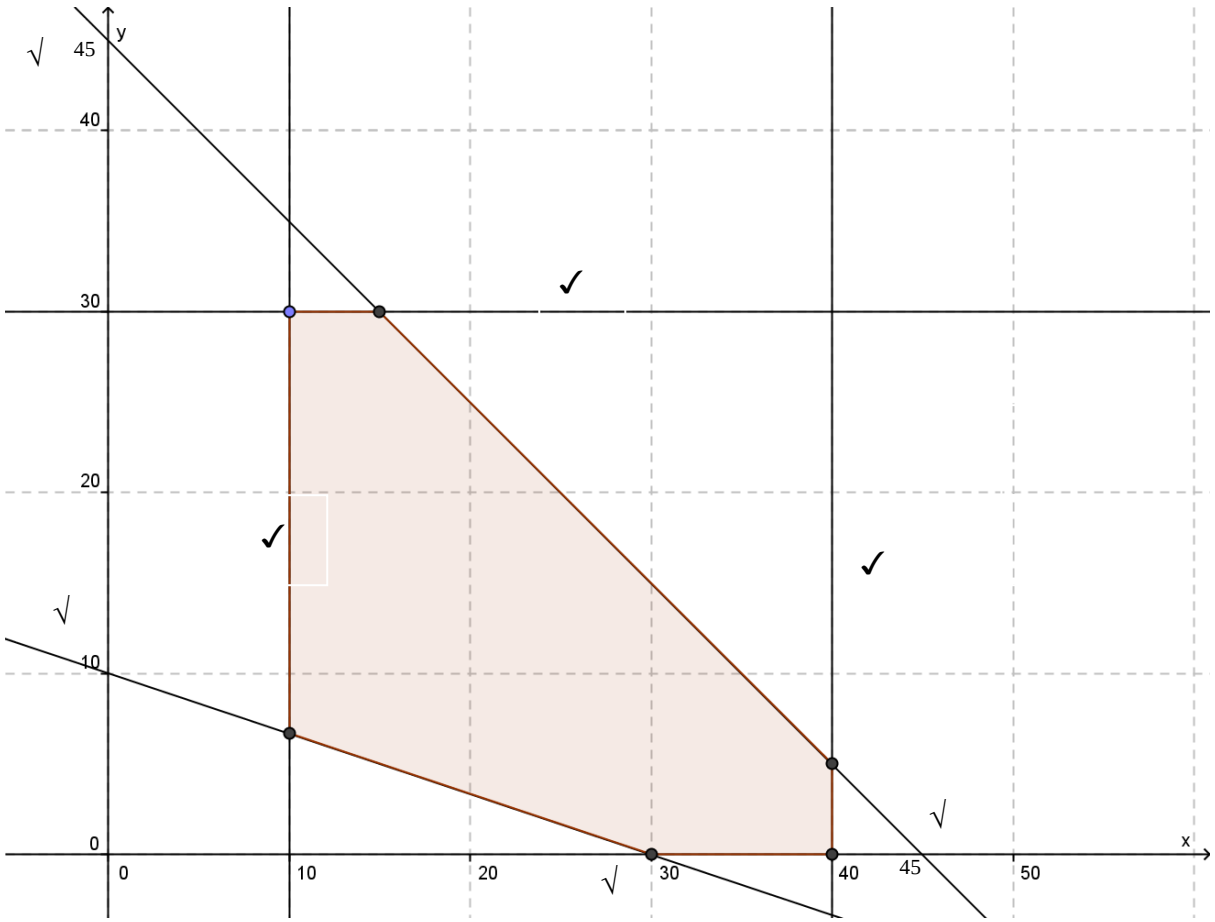
shape✓
x-intercept half each
y-intercept✓
TP✓

(4)

3.2.4 $y \leq 6{,}25$ ✓ $(-\infty;6{,}25]$ (1)

3.2.5 Continuous ✓ (1)

3.3 3.3.1



(5)

3.3.2 Shade feasible region on the graph ✓ (Allow one mistake only) (1)

3.3.3 ✓ ✓
$(x = 15$ & $y = 30)$ Maximum (1 mark for boundary search) ✓
$P_{max} = 0,2x + 0,3y$ ✓
$P = 0,2(15) + 0,3(30)$
$P = 12$ ✓ (Carry forward incorrect *x* and *y*) (4)

3.4 $(2x+1)^2 - 4(x-1)^2 < 0$

$4x^2 + 4x + 1 - 4(x^2 - 2x + 1) < 0$

✓ ✓
$4x^2 + 4x + 1 - 4x^2 + 8x - 4 < 0$

$12x - 3 < 0$ ✓ (Carry forward – one mistake only)

$12x < 3$ ✓

$x < \frac{1}{4}$ ✓

(5)

3.5 $2x^2+5+20x=20-8x^2+x$

$10x^2+19x-15=0$ ✓

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-19\pm\sqrt{(19)^2-4(10)(-15)}}{2(10)}$ ✓

$x=\dfrac{-19\pm\sqrt{961}}{20}$

$x=0.6$ ✓ or $x=-2.5$ ✓ (4)

3.6 $y=5x-26$
$y=x^2-7x+10$

$5x-26=x^2-7x+10$ ✓

$x^2-7x+10-5x+26=0$
$x^2-12x+36=0$ ✓ **(**Carry forward one mistake only)

$(x-6)(x-6)=0$ ✓ **(**Any method - could have used quadratic formula)
✓
$x=6$ and $y=4$ ✓

$(6,4)$ (5)

3.7 $\dfrac{xy-y^2}{x^2-y^2}\times\dfrac{x^2+xy}{y}$

√ √

$\dfrac{y(x-y)}{(x-y)(x+y)}\times\dfrac{x(x+y)}{y}$

✓

$=x$ ✓

(3)

3.8 $\left(x+y^{-1}\right)^{-1}=\dfrac{y}{xy+1}$

LHS $=\left(x+\dfrac{1}{y}\right)^{-1}$ ✓

$=\left(\dfrac{xy+1}{y}\right)^{-1}$ ✓

$=\dfrac{y}{xy+1}$ ✓

$\therefore$ LHS=RHS

Or

$\left(x+y^{-1}\right)^{-1}$

$=\dfrac{1}{\left(x+y^{-1}\right)}$ ✓

$=\dfrac{1}{x+\dfrac{1}{y}}$ ✓

$=\dfrac{1}{\dfrac{xy+1}{y}}$ ✓

$=\dfrac{y}{xy+1}$ ✓

$\therefore$ LHS=RHS

(4)

[50]

TOTAL: 100