higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(PAPER 1)
NQF LEVEL 3

NOVEMBER 2011

(10501053)

8 November (X-Paper)
09:00 – 12:00

This question paper consists of 5 pages, 1 page formula sheet and 1 page annexure.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Write neatly and legibly.
5. Show all working.
6. Question 1.4 and 1.5.1 must be answered on the attached answer sheet (ANNEXURE A), which must be handed in together with the answer booklet.

QUESTION 1

1.1 Solve for x in the following equations:

1.1.1 $x+\frac{6}{x}=5$ (2)

1.1.2 $x^2+(x+2)^2=100$ (3)

1.2 By completing the square solve for x in the following equation:
$7x^2=4x+5$ (4)

1.3 Simplify the following algebraic expressions:

1.3.1 $\frac{3a^2-3b^2}{4a^2+4b^2-8ab}\div\frac{3(b+a)^2}{8(a-b)^2}$ (4)

1.3.2 $\frac{a-3}{a^2-7a+12}-\frac{a+3}{a^2-9}$ (6)

1.4 Sketch the graph of the function $f(x)=\frac{3}{x+1}+2$ on ANNEXURE A. Show all intersections with the axes, as well as the asymptotes. (6)

1.5 A linear programming problem has the following constraints.

Explicit constraints:

$6x+12y\geq 24$
$12x+12y\geq 36$
$2x+y\geq 4$
$x+y\leq 5$

Implicit constraints:

$x\geq 0$
$y\geq 0$

1.5.1 Use the answer sheet provided (Annexure A) to sketch the constraint inequalities, clearly showing the feasible region. (5)

1.5.2 The optimization cost equation is $C=0{,}2x+0{,}3y$. Use the graph constructed in QUESTION 1.5.1 to determine the minimum cost by applying the vertices method. (5)

[35]

QUESTION 2

2.1 Determine the following limit:

$$\lim_{x\to 3}\frac{x^2-x-6}{2x-6}$$ (3)

2.2 Determine the derivative of the following function by applying first principles:

$f(x)=x^2-3x$. (4)

2.3 Determine the derivative of each of the following by using the rule in brackets. [Answers are to be left in simplified form]:

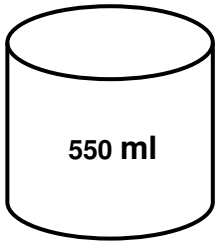
2.3.1 $y=\dfrac{2x^2-4}{2x+3}$ Use the Quotient rule (4)

2.3.2 $y=(2x^2-1)(x+3)$ Use the Product rule (5)

2.4 If $y=\sqrt{(x^2-1)}$, prove that $\dfrac{dy}{dx}=\dfrac{x}{\sqrt{x^2-1}}$. (5)

2.5 Find the equation of the tangent to the curve $y=(x^2+2)(x-1)$ at the point (-2; -3). (4)

2.6 The diagram below shows a closed metal cylinder with a capacity of $550\,ml$. The volume of the cylinder is given by the formula, $V=\pi r^2h$, where r is the radius and h is the height.



2.6.1 Express the height (h) of the cylinder in terms of r. (2)

2.6.2 Show that the surface area of cylinder can be written as $s=2\pi r^2+\dfrac{1100}{r}$ (3)

2.6.3 Calculate the radius of the cylinder with a minimum surface area. (4)

[34]

QUESTION 3

3.1 Simplify the following and write your answer in rectangular form. Show all working.

3.1.1 $(2+3i)+(-i+3)+2i$ (2)

3.1.2 $(3+2i)^2(-1-i)$ (3)

3.1.3 $\dfrac{1}{-i+2}$ (4)

3.1.4 $(3-\sqrt{-5})(5-\sqrt{-10})$ (3)

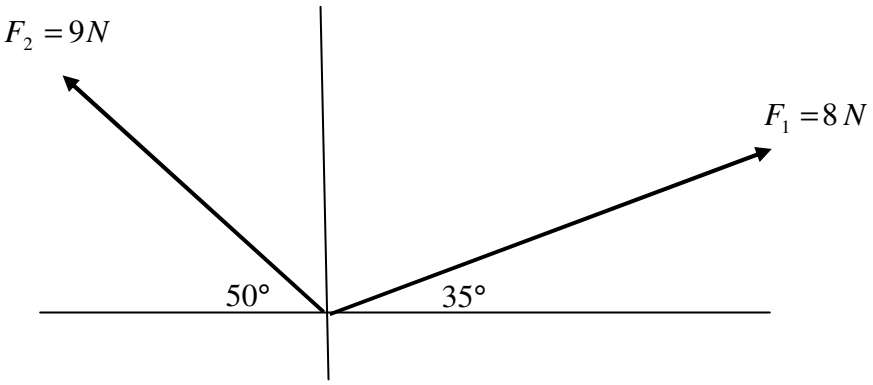
3.2 Use the quadratic formula to solve for the complex roots of $x^2-2x+2=0$, giving your answer in rectangular form. (5)

3.3 Simplify the following:

3.3.1 $\sqrt{(3+4i)}(3+4i)^{\frac{3}{2}}$ (5)

3.3.2 $\dfrac{5\,cis120^\circ.20\,cis60^\circ}{4\,cis30^\circ}$ (4)

3.4 Calculate the resultant force (R) in polar form of the following system of forces acting on a point.



$R = F_1 + F_2$
$F_1 = 8\,cis\,35°$
$F_2 = 9\,cis\,130°$

(5)
[31]

TOTAL: 100

FORMULAE SHEET

1. $Z = r\cos\theta + r\,j\sin\theta$

2. $Z = a \pm bj$ or $Z = a \pm bi$ *where* $i = j = \sqrt{-1}$

3. $r \angle \theta = r\ cis\ \theta$

4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

5. $\dfrac{d}{dx}x^n = nx^{n-1}$

6. $\dfrac{d}{dx}k = 0$

7. $\dfrac{dy}{dx} = u.\dfrac{dv}{dx} + v.\dfrac{du}{dx}$ or $\dfrac{d}{dx}[f(x).g(x)] = f(x)\ g'(x) + f'(x)\ g(x)$

8. $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$ or $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\ f'(x) - f(x)\ g'(x)}{[g(x)]^2}$

9. $\dfrac{dy}{dx} = \dfrac{du}{dx} \times \dfrac{dy}{du}$ or $\dfrac{d}{dx}f\left(g(x)\right) = f'\left(g(c)\right)\ g'(x)$

10. If $y = \ln kx$ then $\dfrac{dy}{dx} = \dfrac{k}{x}$ or If $f(x) = \ln kx$ then $f'(x) = \dfrac{k}{x}$

11. If $y = e^x$ then $\dfrac{dy}{dx} = e^x$ or . If $f(x) = e^x$ then $f'(x) = e^x$

12. If $y = e^{kx}$ then $\dfrac{dy}{dx} = ke^{kx}$ or If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$

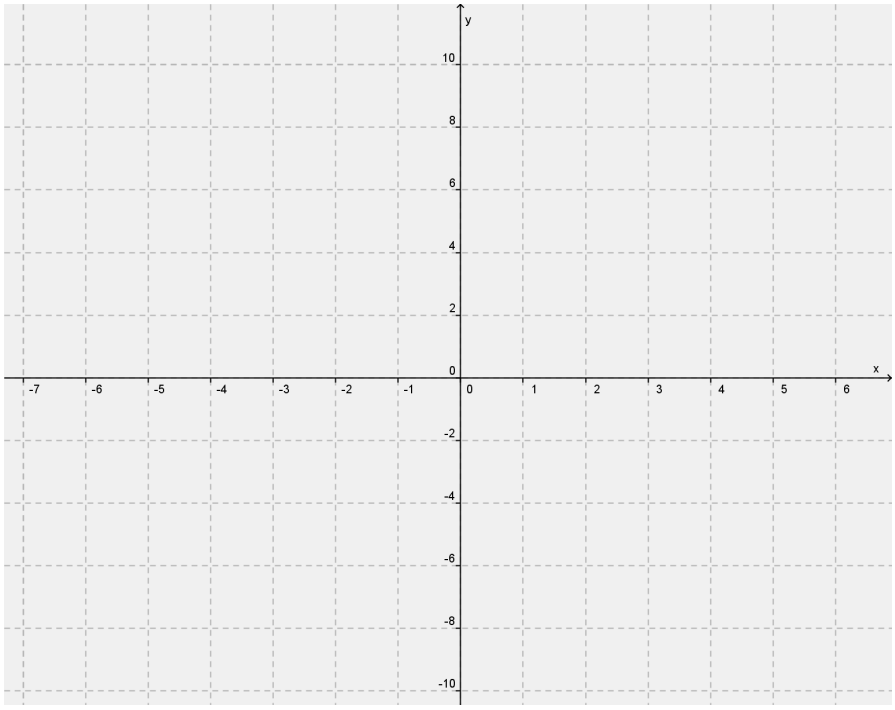
13. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ANSWER SHEET **EXAMINATION NUMBER:**

ANNEXURE A

Detach ANNEXURE A and hand it in with your answer booklet.

1.4



(4)

1.5.1