higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

NOVEMBER EXAMINATION 2011

MATHEMATICS
(PAPER 1)
NQF LEVEL 3

8 NOVEMBER 2011

This marking guideline consists of 11 pages.

Note: Solutions that are different from those in the memo, but are mathematically accurate, shall be accepted.

✓= 1 MARK √ $=\frac{1}{2}$ MARK

QUESTION 1

1.1 1.1.1

$$x+\frac{6}{x}=5$$

$x^2+6=5x$ √ Multiplying by x

$x^2-5x+6=0$

$(x-3)(x-2)=0$ √ Factorizing correctly

√ √

$\therefore x=3 \text{ or } x=2$ Carry forward mistake from previous step (2)

1.1.2 $x^2+x^2+4x+4=100$

$2x^2+4x-96=0$ ✓ Like terms and regrouping

$2(x^2+2x-48)=0$

$(x+8)(x-6)=0$ ✓ Factors

√ √

$x=-8 \text{ or } x=6$ Carry forward mistake from previous step (3)

1.2 $7x^2-4x-5=0$

$7[x^2-\frac{4}{7}x-\frac{5}{7}]=0$ √ Standard form

$x^2-\frac{4}{7}x=\frac{5}{7}$ √

$x^2-\frac{4}{7}x+(\frac{1}{2}\times\frac{4}{7})^2=\frac{5}{7}+\frac{4}{49}$ ✓ Completing the square

$(x-\frac{2}{7})(x-\frac{2}{7})=\frac{39}{49}$ Factors

$(x-\frac{2}{7})^2=\frac{39}{49}$ √

$x-\frac{2}{7}=\pm\frac{\sqrt{39}}{7}$ √

$x=\frac{2}{7}\pm\frac{\sqrt{39}}{7}$ √

$x=\frac{2\pm\sqrt{39}}{7}$ √ Correct solution

$x=-0,606;\ 1,178$

(4)

1.3.1

$$= \frac{3a^2 - 3b^2}{4a^2 + 4b^2 - 8ab} \div \frac{3(b+a)^2}{8(a-b)^2}$$

$$= \frac{3(a^2 - b^2)}{4(a^2 + b^2 - 2ab)} \checkmark \times \frac{8(a-b)^2}{3(a+b)^2}$$ Reciprocal

$$= \frac{3(a+b)(a-b)}{4(a-b)(a-b)} \checkmark \times \frac{8(a-b)(a-b)}{3(a+b)(a+b)}$$ For each correct Factor

Carry forward mistake

$$= \frac{2(a-b)}{(a+b)} \checkmark$$ (4)

1.3.2

$$\frac{a-3}{a^2-7a+12} - \frac{a+3}{a^2-9}$$

$$= \frac{a-3}{(a-4)(a-3)} \checkmark - \frac{a+3}{(a+3)(a-3)} \checkmark$$ Factorising denominator

$$= \frac{(a+3)(a-3) - (a+3)(a-4)}{(a-3)(a-4)(a+3)} \surd \surd \surd$$ Carry forward mistake from previous step

$$= \frac{(a^2 - 3a + 3a - 9) - (a^2 + 3a - 4a - 12)}{(a-4)(a-3)(a+3)} \surd$$ Simplification

$$= \frac{a^2 - 9 - a^2 + a + 12}{(a-4)(a-3)(a+3)} \surd$$ Regrouping

$$= \frac{a+3}{(a-4)(a-3)(a+3)} \surd$$ Numerator

$$= \frac{1}{(a-4)(a-3)} \checkmark$$ Answer (6)

Alternative answer

$$\frac{a-3}{a^2-7a+12} - \frac{a+3}{a^2-9}$$

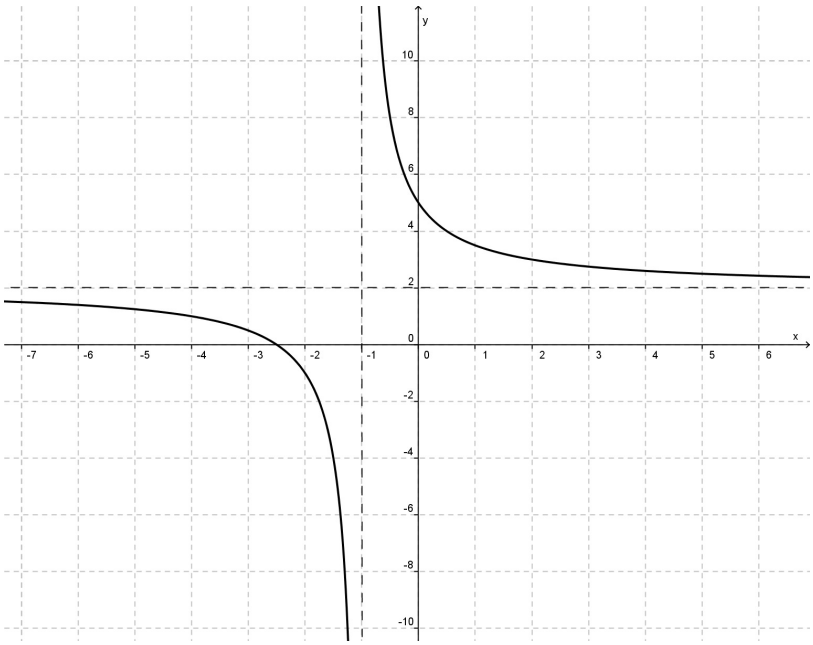
$$= \frac{a-3}{(a-4)(a-3)} \checkmark - \frac{a+3}{(a+3)(a-3)} \checkmark$$ Factorising each denominator

$$= \frac{1}{(a-4)} - \frac{1}{(a-3)} \checkmark$$ Simplifying

$$= \frac{a-3-a+4}{(a-4)(a-3)} \checkmark\checkmark$$ Numerator and denominator

$$= \frac{1}{(a-4)(a-3)} \checkmark$$ Answer

1.4

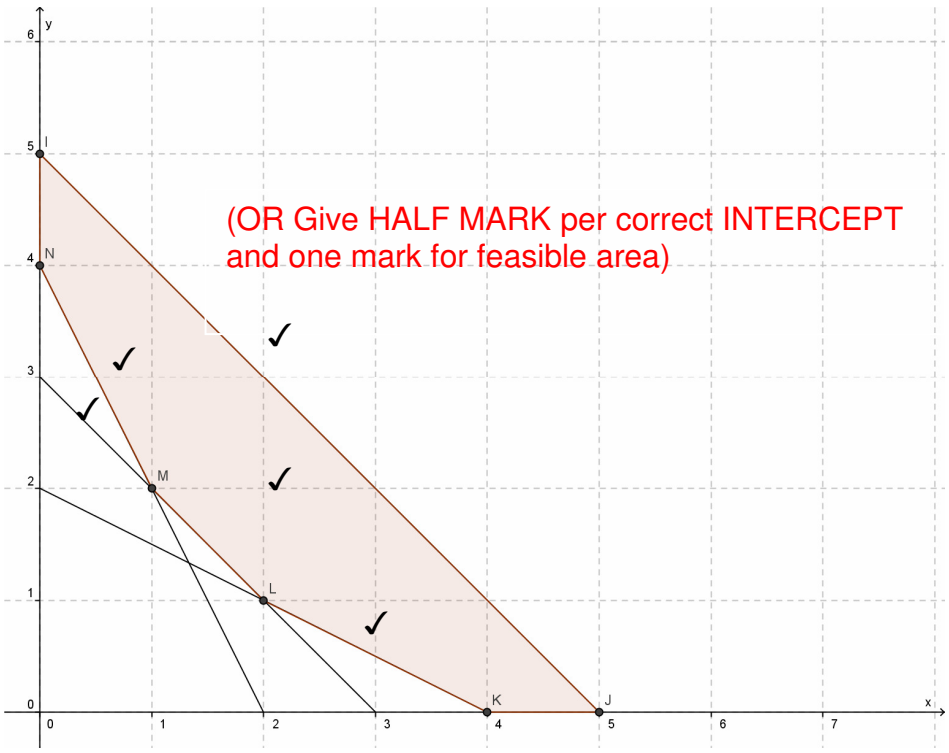


(6)

Allocate marks for intercepts and asymptotes on or off the graph:

x-intercept: $(-2\frac{1}{2}; 0)$	✓
y-intercept: $(0; 5)$	✓
Vertical asymptote: $x = -1$	✓
Horizontal asymptote: $y = 2$	✓
Branches:	✓✓

1.5.1



(5)

1.5.2 $C_M = 0,2x + 0,3y$
$= 0,2(1) + 0,3(2)$ ✓
$= 0,8$ √
$C_L = 0,2x + 0,3y$
$= 0,2(2) + 0,3(1)$
$= 0,7$ √
$C_N = 0,2x + 0,3y$
$= 0,2(0) + 0,3(4)$
$= 1,2$ √
$C_I = 0,2x + 0,3y$
$= 0,2(0) + 0,3(5)$
$= 1,5$ √
$C_J = 0,2x + 0,3y$
$= 0,2(5) + 0,3(0)$
$= 1,0$ √
$C_K = 0,2x + 0,3y$
$= 0,2(4) + 0,3(0)$
$= 0,8$ √
The minimum cost occurs at L = 0,7 ✓ (5)
[35]

✓ One mark for realising they need to substitute into the cost price formula
Work with incorrect coordinates on graph

QUESTION 2

2.1 $\lim\limits_{x\to 3} \dfrac{x^2 - x - 6}{2x - 6}$
$\lim\limits_{x\to 3} \dfrac{(x-3)(x+2)}{2(x-3)}$ ✓ Factorisation
$\lim\limits_{x\to 3} \dfrac{x+2}{2}$ ✓ Simplification
$= \dfrac{5}{2}$ ✓ Correct solution (3)

2.2
$$f(x+h)=(x+h)^2-3(x+h)$$
$$=x^2+2xh+h^2-3x-3h \quad ✓$$
$$m=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h} \quad ✓$$
$$=\lim_{h\to 0}\frac{2xh+h^2-3h}{h} \quad √$$
$$=\lim_{h\to 0}\frac{h(2x+h-3)}{h} \quad √$$
$$=2x-3 \quad ✓$$
(4)

2.3.1
$$y=\frac{2x^2-4}{2x+3}$$
$$√ \quad √$$
$$\frac{du}{dx}=4x;\frac{dv}{dx}=2$$
$$y^1=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{(v)^2}$$
$$√ \quad √$$
$$y^1=\frac{4x\times(2x+3)-2(2x^2-4)}{(2x+3)^2} \quad ✓$$
$$y^1=\frac{8x^2+12x-4x^2+8}{(2x+3)^2} \quad √$$
$$y^1=\frac{4x^2+12x+8}{(2x+3)^2} \quad √$$

Award mark for each correct derivative in numerator if not shown separately above
Substitution
(4)

2.3.2
$$y=(2x^2-1)(x+3)$$
$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$
$$u=2x^2-1 \quad v=x+3$$
$$\frac{du}{dx}=4x \quad ✓ \qquad \frac{dv}{dx}=1 \quad ✓$$

Each correct derivative

$$\frac{dy}{dx}=1(2x^2-1)+4x(x+3) \quad ✓ \quad ✓$$

Substitution

$$\frac{dy}{dx}=2x^2-1+4x^2+12x \quad √$$

Simplification

$$\frac{dy}{dx}=6x^2+12x-1 \quad √$$

Solution
(5)

2.4

$$y=\sqrt{(x^2-1)}=(x^2-1)^{1/2}$$

$Let\ u=x^2-1$

$\frac{du}{dx}=2x$ ✓ Correct derivative

$y=u^{\frac{1}{2}}$ ✓ Substitution

$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}$ ✓ Correct derivative

$\frac{dy}{dx}=\frac{du}{dx}\times\frac{dy}{du}=2x\times\frac{1}{2}(x^2-1)^{-\frac{1}{2}}$ ✓ Substitution

$\frac{dy}{dx}=x(x^2-1)^{-\frac{1}{2}}$ ✓ Solution with final step

$\frac{dy}{dx}=\frac{x}{\sqrt{(x^2-1)}}$

(5)

2.5

$y=(x^2+2)(x-1)$

$y=x^3-x^2+2x-2$ Simplification

$\frac{dy}{dx}=3x^2-2x+2$ ✓ Correct derivative (OR 1 Mark – product rule)

$\frac{dy}{dx}=3(-2)^2-2(-2)+2$ Substitution

$=12+4+2$

$=18$ ✓ Solution

$y-y_1=m(x-x_1)$

$y-(-3)=18[(x-(-2)]$ ✓ Substitution

$y=18x+33$ ✓ Solution (4)

OR

$y=mx+c$

$-3=18(-2)+c$

$33=c$ ✓

$y=18x+33$ ✓

2.6.1

$V = \pi r^2 h$

$V = 550\,ml$ √

$550 = \pi r^2 h$ √

$h = \dfrac{550}{\pi r^2}$ ✓

Solution (2)

2.6.2

Surface area = area of 2 circles + 1 rectangle. Note that the length of the rectangle is equal to the circumference of the circle.

TWO circles $A = 2\pi r^2$

$A(\text{rectangle}) = l \times b = 2\pi rh$ ✓ Correct formula

$= 2\pi r \times \dfrac{550l}{\pi r^2}$ ✓ Correct substitution

$s = 2\pi r^2 + 2\pi r \times \dfrac{550}{\pi r^2}$ ✓ Solution

$s = 2\pi r^2 + \dfrac{1100}{r}$ (3)

2.6.3

$\dfrac{ds}{dr} = 4\pi r - 1100r^{-2}$ ✓ Correct equation

$\dfrac{ds}{dr} = 0 \ \therefore 4\pi r - 1100r^{-2} = 0$

$0 = 4\pi r - \dfrac{1100}{r^2}$ √ Simplification

$0 = 4\pi r^3 - 1100$ √ Simplication

$4\pi r^3 = 1100$ √

$r^3 = \dfrac{1100}{4\pi}$ √ Value r^3

$r = \sqrt[3]{\dfrac{1100}{4\pi}}$ √

$r = 4{,}44$ √ Solution (4)

[34]

QUESTION 3

3.1 3.1.1

$= (2+3i)+(-i+3)+2i$
$= 2+3i+3-i+2i$ ✓ Simplification
$= 5+4i$ ✓ Solution (carry forward mistake) (2)

3.1.2

$= (3+2i)^2(-1-i)$
$= (9+12i+4i^2)(-1-i)$ √
$= (9+12i-4)(-1-i)$ √ Product (carry forward one mistake)
$= (5+12i)(-1-i)$ √ $4i^2 = -4$
$= -5-5i-12i-12i^2$ √ Simplification
$= -5-17i+12$
$= 7-17i$ √ √ Solution (3)

3.1.3

$= \frac{1}{2-i}$
$= \frac{1}{2-i} \times \frac{2+i}{2+i}$ ✓ Conjugate
$= \frac{2+i}{4-i^2}$ ✓ Simplification
Correct denominator
$= \frac{2+i}{5}$ ✓ Solution (4)

3.1.4

$= \left(3-\sqrt{-5}\right)\left(5-\sqrt{-10}\right)$
$= \left(3-\sqrt{5}i\right)\left(5-\sqrt{10}i\right)$ √ √ $\sqrt{-1} = i$ both terms
$= 15-3\sqrt{10}i-5\sqrt{5}i+\sqrt{5\times10}i^2$ ✓ Carry forward mistake
Simplifying
$= 15-3\sqrt{10}i-5\sqrt{5}i-\sqrt{50}$ ✓ Correct solution (3)

3.2 $y = x^2 - 2x + 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \quad ✓$$

Substitution
(carry forward one mistake only)

$$= \frac{2 \pm \sqrt{-4}}{2} \quad ✓$$

Simplifying

$$= \frac{2 + \sqrt{4}i}{2} \quad or \quad \frac{2 - \sqrt{4}i}{2} \quad ✓$$

Each value of x

$$= 1 + i \quad ✓ \quad or \quad 1 - i \quad ✓$$

Each solution (5)

3.3.1 $= \sqrt{(3+4i)}(3+4i)^{\frac{3}{2}}$

$= (3+4i)^{\frac{1}{2}}(3+4i)^{\frac{3}{2}}$ ✓ Exponent of $\frac{1}{2}$

$= (3+4i)^{\frac{1}{2}+\frac{3}{2}}$ ✓ Addition of exponents

$= (3+4i)^2$ ✓ Simplification

$= 9 + 24i + 16i^2$ ✓

$= -7 + 24i$ ✓ Solution (5)

3.3.2 $= \frac{5\,cis\,120° \times 20\,cis\,60°}{4\,cis\,30°}$

$= \frac{5 \times 20}{4} cis\,(120° + 60° - 30^0)$ ✓ ✓ ✓ Correct values and angles

$= 25\,cis\,150°$ ✓ Solution

OR

$= \frac{5 \times 20 cis(120^o + 60^o)}{4 cis 30^o}$ ✓ ✓

$= \frac{100 cis 180^o}{4 cis 30^o}$

$= 25 cis 150^0$ ✓ ✓ (4)

3.4 $R = F_1 + F_2$

$F_1 = 8\,cis\,35° = 6{,}55 + 4{,}58i$ √ √

$F_2 = 9\,cis\,130° = -5{,}78 + 6{,}89i$ √ √

$R = 6{,}55 + 4{,}58i - 5{,}78 + 6{,}89i$

$= 0{,}77 + 11{,}47i$ √ √

$= 11{,}496\,cis\,86{,}15°$ ✓ ✓

Carry forward consistent error (i.e. for correctly changing from standard form to polar form)

OR

$F_1 = 8\,cis\,35° = 6{,}553 + 4{,}589i$ √ √

$F_2 = 9\,cis\,130° = -5{,}785 + 6{,}894i$ √ √

$R = 6{,}553 + 4{,}589i - 5{,}785 + 6{,}894i$

$= 0{,}768 + 11{,}483i$ √ √

$= 11{,}509\,cis\,86{,}173°$ ✓ ✓

(5)
[31]

TOTAL: [100]